Issues in VEDIC MATHEMATICS

Edited by
H.C. KHARE

Rashtriya
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ISSUES IN VEDIC MATHEMATICS

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PROCEEDINGS OF THE NATIONAL WORKSHOP ON
VEDIC MATHEMATICS

25-28 March, 1988
at the University of Rajasthan, Jaipur

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CONTENTS

Contents

Report on the Workshop

A Workshop on Vedic Mathematics was held at the University of Rajasthan, Jaipur, from 25 to 28 March, 1988, under the auspices of Rashtriya Veda Vidya Pratishthan, in collaboration with the Government of India, the Indian Council of Philosophical Research, Rajasthan University, Pondicherry University, and Rashtriya Sanskrit Sansthan. The objectives of the Workshop were to highlight the contributions in the field of mathematics, of books on Vedic Mathematics and allied subjects published during the Vedic period, and also to examine how far such contributions could be incorporated in our present studies of mathematics at the school, college and university levels. The Workshop might also study the form in which Vedic mathematics could be made available to professional mathematicians, computer scientists, and other users of mathematics. Such a study was felt to be important, since the discoveries in respect of mathematical knowledge which are a part of our national heritage should be usefully projected before the younger generation of our country, at different stages of education. The Programme of the Workshop would include, (a) an exposition of Vedic Mathematics by experts in the subject; (b) interaction of these experts with mathematicians, teachers of mathematics, and computer scientists; and (c) a discussion on how Vedic Mathematics could be utilized in the computer science.

These objectives were clearly enunciated in a letter to all the participants by Shri Kireet Joshi, Special Secretary, Department of Education, Ministry of Human Resource Development, New Delhi.

In order to achieve these objectives effectively a diverse cross-section of scholars with varied interests was invited to attend the Workshop. All the delegates were either directly or indirectly connected with and interested in the furtherance of the cause of mathematics, especially ancient Indian mathematics.

The Workshop was organized by University of Rajasthan, Jaipur, under the Chairmanship of Professor R.P. Agarwal, Vice-Chancellor. The inaugural function on 25 March 1988, was presided over by the Honourable L.P. Shahi, Minister of State for Education and Culture, Ministry of Human Resource Development, Government of India, New Delhi.

Dr. Girija Vyas, Minister of State for Education, Government of Rajasthan, was the Chief Guest.

After the inaugural function, Shri L.P. Shahi inaugurated an exhibition of books and rare manuscripts organized by courtesy of the Department of Mathematics and Astronomy, Lucknow University, Lucknow. Shri Shahi and the participants evinced great interest in the publications which were exhibited.

The Workshop was addressed by Professor K.S. Shukla, Dr. N. Puri, Professor W.H. Abdi, Professor C. Santhamma, and Shri Ishwarbhai Patel. The available texts of the talks are here presented. 26 March 1988, the second day of the Workshop, was devoted to Lectures by Professor Ashok Sharma, Professor T.M. Karade, Dr. N. Puri, all of whom excellently expounded different aspects of the Vedic mathematics *sūtras*, their application and effectiveness for simplified calculations. Professor J.L. Bansal also reviewed the works of some of the leading ancient Indian mathematicians. 27 March, 1988, the third day of the Workshop, was devoted to a Lecture by Shri K.C. Kulish, on 'Vedas as Science'. Shri Abhay Kashyap gave a thought-provoking talk entitled 'Some Ramifications'.

Professor Daya Krishna, the philosopher, addressed the delegates on some ramifications of the store knowledge available in the Vedas and their impact on Indian scientific and philosophical thought.

The latter half of the morning session was presided over by Shri Ishwarbhai Patel. All the participants were formally introduced. (A list of all the participants is to be found in Appendix I). The delegates were invited to give further suggestions to implement the objectives of the Workshop. They suggested various ways of propagation of Vedic Mathematics and creation of a general awareness of its value in the delegates was very encouraging and fruitful.

The afternoon session was devoted to lectures on Vedic Mathematics and its propagation and the history of mathematics. Talks were given by Shri S.K. Kapoor, Shrimati Ranjani Chari, Dr. S.A. Paramhans, Shri Dileep Kulkarni, Dr. S.N. Pandey, and Shri Dila Ram.

The Director appointed three groups of delegates to suggest ways and means to achieve various aspects of the objectives of the Workshop as a follow-up measure, in the areas mentioned below:

1. General Awareness of Vedic Mathematics
2. Long-term Plans for Propagation of Vedic Mathematics
3. Curriculum Making at the School, College and University Levels.

The available papers of the abstracts presented at the Workshop have been put together to give a comprehensive idea of the different viewpoints put forth in the Workshop.

Welcome Address

R.P. AGARWAL

Hon'ble Shri Shahi, Respected Dr. Girija, Professor Khare, Dr. Mandan Mishra, Delegates to the Workshop, Ladies and Gentlemen!

It is my proud privilege to welcome you all this morning to the portals of the University of Rajasthan. To Shri Shahi we are particularly indebted that he has been kind enough to accept our invitation to grace this occasion in spite of the fact that the Parliament Session is on. To Dr. Girija Vyas also we are very thankful, although she is a part of this University.

It is a happy augury that this First Workshop on Vedic Mathematics should be held in this historic pink city of Jaipur under the auspices of the University of Rajasthan. The founder of this historic city Sawai Jai Singh II was himself a great Indian astronomer who has left an indelible mark on the history of astronomical sciences in the form of the Jantar Mantar at Jaipur.

Jaipur is a city that glows distinctively in its stroke of pink. Reflecting a spurt of colour, a constant bustle, and a charm of the old world, it makes you feel that kings did live here. The state capital, built in 1727 owes its name to the astronomer prince—Sawai Jai Singh II. Cradled among the rugged hills of the Aravallis, Jaipur is a fairy-tale in stone, chiselled out of the desert. The exquisite buildings in pink terracotta, tracing the delicate filigree work on overhanging balconies, echo the tales of the chivalrous Rajputs, reviving legends, creating a world unfolding a culture — unique and amazing. Jaipur offers a radiance — dazzling with peacocks, palaces, forts, gardens and intriguing bazaars. The silver and ivory intricacies, quaint marble objects and jewellery are spangled all over the bazaar.

It is indeed a fascinating venture to have a Workshop on this rather debatable but imporant topic in the context of our rich cultural heritage. This subject naturally demands the demarcation between the periods of Vedic Mathematics, Post-Vedic Mathematics, Medieval Mathematics and Modern Mathematics. The only scientific way of accomplishing such a subtle and formidable task is to divide the periods on the basis of the time when the various texts were written. The age of the Vedas, itself, has been the subject of extensive and intensive study by numerous scholars in India and abroad. On the basis of the evidence found in certain verses of the

Ṛgveda the age of the Vedas comes to about 2500 B.C. or earlier. A close examination of the various other viewpoints in this respect fixes the age of the Vedas somewhere between 2500 and 500 B.C. Granting this to be the age of the Vedas, we can conveniently divide the later periods on the basis of the nature of other available works.

The history of evolution of Vedic Mathematics indicates that it has been more than often a very sensitive area, whose growth has been embedded in the passions and prejudices of sectarian jealousy and fanaticism, by introducing the sensitive issue of religion. It is in this context that the present workshop is very significant and we have to present the diverse opinions in the correct modern perspective, without bringing in the conflicting socio-religious philosophies and institutions. We know that the Vedas consist of a vast treasure of supreme knowledge not to be found in the world during the period they were supposed to have been written. The *Atharvaveda*, particularly, according to Jagadguru Svāmī Śrī Bhāratī Kṛṣṇa Tīrthaji Mahārāja, has a collection of a number of *sūtras* which cover a wide range of simplified methods of computations in arithmetic, algebra, geometry and elementary calculus. The ingenuity of using these *sūtras* lies in their correct interpretation and usage.

In the later period, from 500 B.C. up to A.D. 500, one comes across the *Arthaśāstra* of Kauṭilya and the *Sūrya-prajñapti*, representing Jaina astronomy. These books show no further improvement in the astronomical constants as given earlier in the *Vedāṅga Jyotiṣa*. In the Jaina and Pauranic works, we find the theory of flat earth, the sun, moon and star, moving in circles around the pole of the earth. The *Āryabhaṭīya* of Āryabhaṭa I, written in A.D. 499 marks the beginning of purely scientific astronomy which continues right up to the end of the eighteenth century.

The Vedic Hindus particularly evinced special interest in geometry (*Śulba*) and astronomy (*Jyotiṣa*). Probably the reasons for this have their origin in the philosophy of our ancient culture. It is very astonishing that without the modern scientific equipments they insisted on extreme accuracy and rigour in all their findings and endeavours. In geometry, Baudhāyana (800 B.C.) expressly stated the so-called Pythagora's theorem and utilized this theorem, as well as its converse, in the constructions of various geometrical figures. There are, in *Śulba*, solutions of simultaneous indeterminate equations also. Of course, these equations had their origin in the formation of various kinds of fire-altars, to be constructed from bricks, in connection with religious ceremonies.

The Āyurveda consists of mostly verses connected with experimental researches dealing with life in all its phases—philosophical and biological and comprises both preventive and curative medicine and surgery. The *Sāmaveda* is a collection of musical Sāman hymns recited at the time of Vedic rituals.

I have every hope, that in spite of the fact that the cultural metamorphosis of the Indian mind because of the Western progress has been so progressive and fast during the last six or seven centuries, one does not tend to look with reverence at our ancient culture and the devoted endeavours of the great hermits. We shall, through discussions during a workshop like this, awaken the necessity of restoration and development of the scientific content of the Vedas, for the benefit of our country. It has not to be done merely as an act of patriotism but in a spirit of modern scientific enquiry and we should try to correlate the mystic past with the modern scientific temper.

Let this day mark a beginning and promise for further, for making such a sacred effort.

I once again thank Hon'ble Shri Shahi and Dr. Girija Vyas for kindly gracing the function.

Vedic Mathematics

H.C. KHARE

Being no Vedic scholar I feel it an honour to be asked to be the Director of the Workshop on Vedic Mathematics, although I would have liked it to be called 'Mathematics in the Vedas'. The subject of Vedic Mathematics has come in sharp focus after the publication of the book *Vedic Mathematics* by Svāmī Śrī Bhāratī Kṛṣṇa Tīrtha, Śaṅkarācārya of Govardhana Matha. In his book he has expounded 16 *sūtras* which are claimed to be in the Appendix (*Pariśiṣṭa*) of the *Atharvaveda*. There are thirteen sub-*sūtras* also. These *sūtras* lead to extremely fast mental calculation and can be used in the preparation of computer software which are likely to reduce the computational time. The *sūtrtas* themselves are terse and need expounding and repeated use makes them simple. It is fascinating to note that mostly non-professional mathematicians have taken to Vedic Mathematics calculation methods and an atmosphere has been created in which people are asking more and more questions and want to know Vedic Mathematics. It is, therefore, but imperative to go into Vedic Mathematics and Vedic science, the *sūtras*, their origin, justification, and explanation, and if possible, rewrite the *sūtras* in a logical manner, capable of further investigation by mathematicians and scientists. In this context allow me to give a brief account of science in the Vedas as I have tried to comprehend, which I shall give in Hindi.

'वेद' शब्द का अर्थ है 'ज्ञान'। आज से लगभग 6-7 हजार वर्ष पहले आर्यों ने अपनी दिव्य प्रतिभा से जो ज्ञान प्राप्त किया था, उसे काव्यात्मक भाषा में उपनिबद्ध करके वेद की संज्ञा दी गई। ये वेद जगत के जड़ एवं चेतन रूपों की रहस्यमयी व्याख्याएँ हैं। मूलतः तो वेद एक ही है, किन्तु गद्यपद्यात्मक शैली के आधार पर वर्गीकरण करके छन्दोमयी वैदिक रचना को 'ऋच्' गद्यात्मक वैदिक रचना को 'यजुष्' और गीत्यात्मक रचना को 'सामन्' कहा गया है। इसीलिए वेदों का दूसरा नाम 'त्रयी' भी है। वेदव्यास नामक ऋषि ने साधारण मनुष्यों की सुविधा की दृष्टि से वेद को 'ऋग्वेद', 'यजुर्वेद', 'सामवेद' और 'अथर्ववेद' नामक चार रूपों में विभक्त (arrange) करके प्रस्तुत किया है।

यद्यपि इन वेदों का मुख्य प्रतिपाद्य सम्बोधन, एवं स्तुतिगान ही है, तथापि आनुषंगिक रूप से यत्र-तत्र अन्य समस्त ज्ञान-विज्ञानों के बीज भी उपलब्ध होते हैं। सामाजिक जीवन की विविध झांकियों के अतिरिक्त अन्य प्रकार की सूक्ष्मातिसूक्ष्म जानकारियों के छुट-पुट

संकेत भी वेदों में दृष्टिगोचर होते हैं। यज्ञ के अतिरिक्त अन्य बातों की जानकारी सर्वाधिक अथर्ववेद में प्राप्त होती है। उसमें भैषज्य मन्त्र, अभिचार मन्त्र, मारणमोहन, मन्त्र, आध्यात्मिक एवं दार्शनिक मन्त्र तथा प्रायश्चित्तादि मन्त्र बहुतायत से देखने को मिलते हैं। साथ ही प्रसंमतः आये हुये वैज्ञानिक तथ्यों का भी साक्षात्कार अवश्य होता है। वेदों के सहायक ग्रन्थों के रूप में विकसित हुए शिक्षा, कल्प, व्याकरण, निरुक्त, छन्दःशास्त्र, और ज्योतिष नामक छहों वेदांगों में तो वैज्ञानिक विधियों का और अधिक ज्ञान प्राप्त होता है।

किन्तु इसका अभिप्राय यह नहीं है कि आज का प्रचलित गणित, भौतिकी, रसायन-शास्त्र, जीव-विज्ञान, वनस्पति-विज्ञान, आदि वेदों में पूर्णतया विकसित रूप में वर्णित या व्याख्यात हैं। ऐसा मानना अतिवाद मात्र हैं। इस सन्दर्भ में जगद्गुरु भारती कृष्ण तीर्थ विरचित ग्रन्थ "वैदिक मैथमेटिक्स" या महामहोपाध्याय गिरिधर शर्मा चतुर्वेद कृत "वैदिक विज्ञान और भारतीय संस्कृति" जैसे ग्रन्थों के अवलोकन से वैदिक ऋषियों की वैज्ञानिक दृष्टि का चमत्कारपूर्ण वैशिष्ट्य अवश्य सामने आता है। किन्तु एक शास्त्र में क्रमबद्ध कोई भी विज्ञान, वेदों में ढूँढना, न तो उपादेय है और न ही युक्ति-संगत।

फिर भी जो कुछ छुट-पुट सन्दर्भ इन विज्ञानों के वेदों में देखने को मिलते हैं, उनका संक्षिप्त विवरण इस प्रकार हैः

(1) अथर्ववेद के पृथ्वीसूक्त में पृथ्वी (Earth), भूमि (soil), उसकी विभिन्न परतों (layers), उसकी रत्नधारण क्षमता, उसके विविध रंग—वभ्र (brown colour), कृष्ण (black colour), रोहित (reddish colour), तथा उसकी उत्पादन क्षमता आदि (यस्यामन्नं कृष्टयः संबभूवुः) का सटीक वर्णन किया गया है। वह इन पदार्थों को जिलाने और पुष्ट करने की क्षमता रखती है (यस्यामिदं जिन्वति प्राणदेजत्)। वह ऊर्जा की स्रोत, सक्रियता एवं सजीवता की आश्रय कही गयी है। पृथ्वी में अग्नि, जल और आकाश (fire, water, and space) अन्तर्निहित बताये गये हैं। पूषन् नामक देवता कृषि और पशुओं के स्वामी बताये गये हैं। वे सूर्य के पूर्वगामी देवता हैं। वे उनकी किरणों को धरती के जीवों और वनस्पति पर किस प्रकार अनुप्राणित करते हैं—इस तथ्य का भी आकलन किया गया है।। भूमि को उर्वरा तथा अनुर्वरा बनाने वाले कीटाणुओं की ओर भी निर्देश किया गया है (अथर्ववेद 12.1.46)। भूमिको घृत, मधु,दुग्ध आदि से समन्वित बताया गया है। साथ ही मल्व (waste) को भी भूमि में उर्वरा शक्ति (fertility) बढ़ाने वाला कहा गया है (अथर्ववेद 12.1.48)।

(2) अथर्ववेद में अनेक स्वास्थ्यवर्द्धक एवं रोगनाशक औषधियों का सजीव वर्णन किया गया है, जो कुष्ठ, क्षय एवं ज्वर आदि रोगों को नष्ट करने में अद्भुत सामर्थ्य रखती हैं। ऋग्वेद (1.116.19 और 1.157.6) में भी स्वास्थ्य देने वाली आयु को दीर्घ करने वाली औषधियों की संस्तुति की गयी है। अथर्ववेद का प्रथम काण्ड पूरी तरह से औषधियों (medicines) और रोगों (diseases) के वर्णन से परिपूर्ण है।

प्रसूतितन्त्र (Gynaecology) का भी विशद प्रतिपादन प्रथम काण्ड के पंचम सूक्त में किया गया है। इसी काण्ड के सत्रहवें सूत्र में धमनियों तथा बाइसवें सूक्त में जिगर तथा हृदय (liver and heart) के रोगों का हवाला दिया गया है। इतना ही नहीं प्रथम से लेकर ग्याहरवें काण्ड तक शरीर संरचना, उसके दोष एवं निदानों का विस्तृत किन्तु समुचित वर्णन चलता रहता है। A great scholar has rightly remarked: 'On the whole it might be said safely that Atharva-vedic hymns contain within themselves a large amount of medical and scientific know-how.'

(3) "कल्प" नामक वेदांग के अन्तर्गत विरचित बोधायन शुल्बसूत्र एवम् आपस्तम्ब शुल्बसूत्र आदि ग्रन्थों में ज्यामितीय ज्ञान की निर्भ्रान्ति स्वरूप दर्शनीय है। इसी प्रकार आचार्य लगध प्रणीत वेदांग ज्योतिष में अनेक गणितीय सिद्धान्तों का शुद्ध स्वरूप देखने को मिलता है।

The *sūtras* in the *Atharvaveda* are terse and need elucidation before they can be made use of by working mathematicians. It is clear that they are science as a whole. Here by science is meant knowledge. There is no partition of knowledge into various compartments. It is interdisciplinary. The division of knowledge into various heads is of recent origin. For example, in the western universities one had Natural Philosophy and its demarcation into branches of knowledge is a later development. If Vedic science is made available to ordinary learner, there is a possibility of faster growth and better perception of knowledge. It would also remove the myths that surround Vedic science. It is high time that all that is in the Vedas is explained in a manner which is acceptable to the present learner and I am sure a feeling can be created that there is so much to learn from our Vedas in various fields of knowledge. This is the purpose and the objective of this Workshop on Vedic Mathematics. It is hoped that the scholars assembled here would give serious thought to it and prepare a plan of action to achieve the objectives.

In this connection one should glance at the available material about mathematical knowledge in India. The Hindus had a knowledge of astronomy which they developed to know the recurrence of seasons. They had a calendar based on the sun and the moon and movements of these bodies were carefully recorded over a long perod. There is evidence of acquisition of considerable skill in computaiton. Various historians have observed that the Vedic age exhibits nothing of a mathematical nature. In the absence of proper texts no correct assessment is possible. It is evidenced that the Hindus had aptitude for arithmetical and algebraical aspects of mathematics and these branches were cultivated with zeal. They

had the genius of calculation and reducing these to number zero rules. It appears that the Hindu mathematicians scribbled rules with perfection without emphasizing the methods.

The Hindu contributions in mathematics are significant.Their grandest achievement is the principle of place value which of all the mathematical inventions has contributed most. Their skill in handling problems and equations of more than one variable was unique. Their method of solution of quadratic equations should find a place in modern textbooks. The solution of cubic and quartic led to the modern development in the treatment of these equations. The Hindus unhesitatingly accepted irrational number as the solution. The immensely important concept of absolute negative is due to the Hindus. The most outstanding contribution was the study of indeterminate equation. Here they made some of the discoveries of modern algebra going beyond Diophantus. As pointed out earlier, the study of astronomy led to the *siddhāntas* (established conclusions). The *siddhāntas* were five in number, of these the *Sūrya-siddhānta* and the *Pauliśa-siddhānta* are the most important. These in themselves laid the foundation of Hindu trigonometry. During A.D. 500-1000 India produced four or five mathematicians of repute. The period around the astronomer-mathematician Āryabhaṭa was the most flourishing. Āryabhaṭa systematized the result in the *siddhāntas* and his treatise *Āryabhaṭīya* is of particular value giving the state of mathematical knowledge at that time. It consists of sixty-six rules, many of them extremely complex and difficult to follow. Then followed the period of Brahmagupta, Mahāvīra, Śrīdhara and Bhāskara. Brahmagupta was one of the greatest mathematicians that the country has produced. His work was of original nature and his clear concept of the negative numbers and the zero is clearly perceptible. The concept of the zero is India's contribution to mathematics. The study of intrmediate equation, quadratic, cubic, and quartic equation find important place in his book. The study of progressions and the calculation of ratio of circumference of the circle to its diameter finds a detailed discussion. The knowledge of geometry is exhibited. The problems are difficult and their solutions, and the rules leading to them are given without explanation. The area of a triangle or the square root of complicated calculations is worked out by rules not properly explained. Bhāskara's comprehensive work *Siddhānta-śiromoṇi* has as its first chapter 'Līlāvatī' and the third 'Gaṇita', which together form the most complete exposition of Hindu mathematics. Bhāskara was an outstanding scholar.

This gives ample proof of Indians being in the forefront of mathematics and its development. Considerable evidence is available about ancient India's excellence in fields like medicine and other sciences, besides non-science branches of knowledge. All this has to be made available in a comprehensible manner.

This Workshop has been organized keeping all this in view. Lectures giving exposition to mathematics and sciences and knowledge in the Vedas will be heard during the Workshop. The discussion groups will examine various aspects of Vedic science and mathematics. It will be our endeavour to investigate and recommend what aspects of this knowledge can be incorporated in our textbooks, to make it easily accessible, and to help generate in the learner an awareness of the achievements of Vedic science. The recommendations will be followed by an action plan.

उद्घाटन भाषण

सम्माननीय श्री ललितेश्वर प्रसाद शाही

(शिक्षा और संस्कृति राज्यमंत्री, भारत सरकार)

राजस्थान की संस्कृत शिक्षा मंत्री बहन डॉ. गिरिजा व्यास, राजस्थान विश्वविद्यालय के कुलपति डॉ. रत्न प्रकाश अग्रवाल, राष्ट्रिय संस्कृत संस्थान के निदेशक डॉ. मण्डन मिश्र, कार्यशाला के निदेशक डॉ. खरे, देश के भिन्न-भिन्न भागों से समागत विद्वानों, भाइयों और बहनों!

मानव संसाधन विकास मंत्रालय, भारतीय दार्शनिक अनुसंधान परिषद, राष्ट्रिय वेद विद्या प्रतिष्ठान, राष्ट्रिय संस्कृत संस्थान और राजस्थान विश्वविद्यालय के संयुक्त तत्वाधान में आयोजित कार्यशाला के उद्घाटन के लिये मुझे आमंत्रित किया गया, इसके लिये मैं विश्वविद्यालय के कुलपति डॉ. अग्रवाल के प्रति कृतज्ञता प्रकट करता हूँ।

राजस्थान का त्याग और बलिदान पूर्ण इतिहास हम सबके लिये प्रेरणा का विषय रहा है। विद्या के क्षेत्र में इस प्रदेश का योगदान भी बहुत महत्व रखता है। प्राचीन काल की रियासतों के राजा-महाराजाओं से लेकर हमारे वर्तमान राजस्थान शासन तक ने भारतीय विद्याओं और कला के विकास के लिये जो कार्य किये हैं, उनकी सदा से प्रशंसा होती रही है। विशेषकर वेद और ज्योतिष के क्षेत्र में जयपुर नगरी की बहुत प्रसिद्धि है। यहाँ का ज्योतिष यंत्रालय एक उत्तम वेधशाला के रूप में विख्यात है। वैदिक-विज्ञान के क्षेत्र में एक शताब्दी से कुछ ही वर्ष पूर्व स्वनामधन्य विद्यावाचस्पति श्री मधुसूदन ओझा और उनके शिष्यों ने वेद की वैज्ञानिक व्यवस्थाएँ प्रस्तुत कर सारे संसार को चमत्कृत किया है। यही कारण है कि हमने वेद-विज्ञान से सम्बद्ध एक प्रमुख अंग वैदिक गणित पर कार्यशाला के आयोजन के लिये जयपुर को चुना। राजस्थान विश्वविद्यालय जैसे सुप्रतिष्ठित विश्वविद्यालय ने इस प्रदेश की परम्परा के अनुसार अतिथिसत्कार का दायित्व अपने ऊपर लिया। देश के भिन्न-भिन्न भागों से आप जैसे चुने हुए विद्वान उपस्थित हुए। मैं इन समस्त उपलब्धियों को हमारे एक महान कार्यक्रम के लिये मधुर मंगलाचरण के रूप में स्वीकार करता हूँ और भारत के शिक्षा तथा संस्कृति राज्यमंत्री के रूप में आप सबका स्वागत तथा अभिनन्दन करता हूँ।

वेद इस राष्ट्र की सर्वोत्तम धरोहर है। वेदों की रचना, अपौरुषेयता और काल के संबंध में अनेक धारणायें हैं—लेकिन यह निर्विवाद सत्य है कि वेद संसार के प्राचीनतम ग्रन्थ

हैं। वैदिक वाङ्मय के अध्ययन के अभाव में भारतीय संस्कृति अध्यात्मिक तत्व-ज्ञान को समझ सकना कठिन है। वेद का अर्थ है—ज्ञान अर्थात् ज्ञान का मूल स्वरूप और भारतीय ज्ञान परम्परा का आदिस्रोत है। केवल आध्यात्मिक ज्ञान ही नहीं, अपितु मानव जाति के प्राचीन इतिहास और रहन-सहन, आचार-व्यवहार की जानकारी के लिए भी वह उतना ही उपादेय है। वेदों के शुद्ध पाठ की रक्षा के लिये वैदिक मनीषियों ने क्रमपाठ, जटापाठ, शिखापाठ, घनपाठ आदि विविध पाठों का विधान किया—जिनकी संरचना अत्यन्त वैज्ञानिक है। यही कारण है कि आज भी वेद उसी विशुद्धि तथा प्रामाणिकता के साथ उपलब्ध हो रहे हैं। संसार के साहित्य में यह अत्यन्त विलक्षण उदाहरण है।

वैदिक ज्ञान-पद्धति में मेधा शक्ति के उत्कर्ष के सहज ही दर्शन होते हैं, साम-गायन पद्धति तो वस्तुतः भारतीय संगीत शास्त्र का मूल है। साम-गायन पद्धति के रहस्य का ज्ञान अत्यन्त दुरूह है तथा साम-गायन को ठीक स्वरों में गाने वाले मनीषी ऋषि अब बहुत कम मिलते हैं। भारतीय संगीत-शास्त्र के मूल स्रोत साम-गान की इस पद्धति की रक्षा हमारा प्रथम कर्तव्य हो गया है।

अथर्ववेद का पृथ्वीसूक्त महनीय राष्ट्रीयता का संदेशवाहक बनकर आज भी हमारे लिये उत्साह तथा उल्लास का प्रेरक है। माता के रूप में पृथ्वी की कल्पना अत्यन्त हृदयग्राही है। साथ ही वेद में राष्ट्र की संकल्पना से तत्कालीन समाज के सांस्कृतिक उत्थान के स्पष्ट प्रमाण मिलते हैं—"अहं राष्ट्री संगमनी वसूनाम्"।

ईश्वरीय ज्ञान तथा विज्ञान को एक स्थान पर प्रस्तुत करने वाला साहित्य ही वैदिक वाङ्मय के नाम से अभिहित किया जाता है। वेदों में अध्यात्मवाद और रहस्यवाद के अतिरिक्त वैज्ञानिक तत्वों का भी प्रतिपादन मिलता है। शतपथ-ब्राह्मण में इस सृष्टि की उत्पत्ति के वर्णन से स्पष्ट उल्लेख है कि—"यजुः" "यत्" और "जूः" दो शब्दों से बना है। "यत्" का अर्थ है—निरन्तर चलनशील और "जूः" का अर्थ है स्थिर। शतपथ के अनुसार इन्हीं दोनों तत्वों से सम्पूर्ण सृष्टि की रचना हुई है। आधुनिक विज्ञान भी यह स्वीकार कर चुका है कि मौलिक तत्व केवल दो ही हैं—इलेक्ट्रॉन और प्रोटोन। इनमें से एक तत्व स्थिर है और दूसरा निरन्तर उसके चारों ओर घूम रहा है। अथर्ववेद ऐसे वैज्ञानिक रहस्यों से परिपूर्ण है। स्पष्टतः विज्ञान की ऐसी आधुनिकतम उपलब्धियों का विलक्षण सम्बन्ध श्रुति से है।

वैदिक साहित्य अत्यन्त विशाल है—इसमें निहित ज्ञान की विभूति देवाराधना या विविध संस्कारों तक ही सीमित नहीं है—ज्ञान के आधुनिकतम आयामों के मूल बीज हमें वेदों से प्राप्त हुए हैं। आज आवश्यकता है इन्हें सही रूप में समझने की—वैदिक ऋषियों के मन्तव्यों को जानने के लिये उसकी आत्मा तक पहुँचने की। तभी हम अपने गौरव पूर्ण

अतीत से सही मायनों में जुड़ सकेंगे और आधुनिक विज्ञान के युग में उसके महत्व को उच्च उद्घोष पूर्वक स्थापित कर सकेंगे।

संसार के महान समीक्षकों ने वेद की इस महत्ता पर अनेक रूपों में प्रकाश डाला है। सभी चिन्तकों ने यह स्वीकार किया है कि वेद आध्यात्मिक ज्ञान के साथ-साथ विज्ञान के भी भंडार हैं। अनेक इतिहासकार लिखते हैं कि हमारे चारों वेदों के साथ चार उपवेद जुड़े हैं। ऋग्वेद के उपवेद आयुर्वेद में शरीर शास्त्र के सम्बद्ध आयुर्विज्ञान भरा पड़ा है। यजुर्वेद का उपवेद धर्नुवेद है—जिसमें युद्धशास्त्र का वर्णन है और सेना की रचना, व्यवस्था, विभिन्न प्रकार के आयुधों और शस्त्रों का विस्तार से विवेचन है। सामवेद का उपवेद गन्धर्ववेद है, जो संगीत, नृत्य और अन्य ललितकलाओं के ज्ञान से पूर्ण है। अथर्ववेद का उपवेद स्थापत्यवेद है—जिसमें वैदिक गणित जैसे गम्भीर विषयों का समावेश है। वास्तव में यह सब तो विषयों का स्थूल विभाजन है। मूल रूप से अनेक विषयों का सूक्ष्मतम आधार वेद में है और उसके प्रत्येक क्षेत्र में सूक्ष्म अनुसंधान की आवश्यकता है।

भारत सरकार ने वेद-विद्या की इस गरिमा को स्वीकार किया है और वेद-विद्यालयों के संचालन, वैदिक विद्वानों के सम्मान आदि की दृष्टि से दो दशकों से उल्लेखनीय कार्य कर रही है। लेकिन यह अनुभव किया गया है इस महान ज्ञानराशि के लिये जो प्रयास किये जा रहे हैं, उन्हें और आगे बढ़ाने की आवश्यकता है।

स्वर्गीया प्रधानमंत्री श्रीमती इन्दिरा गांधी जी ने इस दिशा से प्रेरणा प्रदान की, इसके परिणामस्वरूप गत वर्ष सप्तम पंचवर्षीय योजना में डेढ़ करोड़ रुपये की संरक्षित-निधि का प्रावधान कर राष्ट्रिय वेदविद्या प्रतिष्ठान की स्थापना की गई। हमारा यह सौभाग्य है कि हमारे मनीषी मानव संसाधन विकास मंत्री श्री पी.वी. नरसिंहराव ने प्रतिष्ठान की अध्यक्षता स्वीकार की। इसके उपाध्यक्ष के रूप में मुझे इसकी सेवा का अवसर मिलता है। प्रतिष्ठान का मूल उद्देश्य एक ओर वेद का अक्षुण्ण पारम्परिक स्वरूप की रक्षा है और दूसरी ओर इसमें निहित विज्ञान के तत्वों की खोज करना है। पारम्परिक उच्चारण की दिशा में हमारे वेद-विद्यालय काम कर रहे हैं और अब हम वैदिक विज्ञान के क्षेत्र में काम करने को सोच रहे हैं—जिसका प्रथम सोपान आप सब लोगों का समागम है।

प्रतिष्ठान की स्थापना से सारे देश का ध्यान वेद की ओर गया। हम इसे अपनी सफलता का शुभारंभ मानते हैं। लोकसभा और राज्यसभा के अनेक माननीय सदस्यों तथा विदेशी विद्वानों ने कम्प्यूटर के इस युग में वैदिक गणित की उपादेयता के सम्बन्ध में हम सबका ध्यान आकृष्ट किया है। यह सब एक शुभ-लक्षण है।

वैदिक गणित हमारे लिये नया विषय नहीं है। जगन्नाथ पुरी के यशस्वी शंकराचार्य वंदनीय स्वामी भारतीकृष्ण तीर्थ ने बहुत पहले इस ओर संसार का ध्यान आकृष्ट किया।

स्वामी जी का पूर्वाश्रम का नाम वेंकटरामन् था और वे आन्ध्रप्रदेश में राजमहेन्द्री स्थान में उत्पन्न हुए थे। प्राचीन और अर्वाचीन विद्याओं में निष्णात होने के साथ-साथ महान् तत्वज्ञानी और साधक थे। उन्होंने भारत के अनेक विश्वविद्यालयों में वैदिक गणित पर भाषण दिये और सन् 1958 ई. की अपनी विदेश यात्रा में टेक्नोलॉजिकल इंस्टीट्यूट ऑफ केलिफोर्निया (अमरीका) में गणित के विद्वानों के सामने इस विषय का विस्तार से विवेचन किया—जिसे सुनकर वहाँ के विद्वान आश्चर्यचकित रह गये। स्वामी जी ने 1950 ई. में काशी हिन्दू विश्वविद्यालय और 1952 ई. में नागपुर विश्वविद्यालय में इस विषय की शिक्षा एवं दीक्षा दी। उनके प्रतिपादन के अनुसार यह पद्धति अपने आप में अनूठी है और इस पद्धति से प्रश्न बहुत आसानी से हल किये जा सकते हैं। हिन्दू विश्वविद्यालय ने उनके इस ग्रंथ का प्रकाशन किया—जिसके नवीनतम रूप की एक-एक प्रति आपको भेंट की गई है। परम श्रद्धेय स्वामी भारतीकृष्ण तीर्थ जगद्गुरु शंकराचार्य के रूप में श्रद्धा के पात्र थे। वे महान् देशभक्त थे, इसलिए उनका प्रतिपादन मूल रूप में महत्वपूर्ण हो, यह स्वाभाविक है। इसके विषय में बहुत सी आलोचना, प्रत्यालोचना भी हुई है, जो जिज्ञासुओं के लिए अधिक आकर्षण पैदा करती है। आज यूरोप में यह मान्यता प्रकट की जा रही है कि कम्प्यूटर प्रणाली में वैदिक गणित का महत्वपूर्ण स्थान है। इसमें एक-एक प्रश्न के हल करने के कई-कई सरल तरीके हैं और एक पद्धति से प्रश्न करके दूसरी पद्धति से उसे "चैक" करने की भी सुविधा है।

इन सारे सिद्धांतों और मान्यताओं का वैज्ञानिक प्रक्रिया द्वारा परीक्षण कर ही इनकी पुनः स्थापना की जा सकती है। इसी दृष्टि से इस कार्यशाला का आयोजन किया जा रहा है। मेरा विश्वास है कि आप जैसे विद्वान इसकी गहराई में जायेंगे और जो चिन्तन इस दिशा में चल रहा है, शास्त्रीय दृष्टि से उसका मूल्यांकन करेंगे। जैसा कि मैंने ऊपर कहा है कि इस दिशा में हमारा यह पहला प्रयास है। वेद तो विज्ञान का भंडार है, हम आगे जाकर वैदिक-विज्ञान के अन्य क्षेत्रों पर भी विचार करने का प्रयास करेंगे। आशा है कि आप जैसे प्राच्य और प्राचीच्य विद्वानों का सहयोग हमें प्राप्त होगा। इन्हीं शब्दों के साथ मैं इस कार्यशाला का उद्घाटन करता हूँ।

धन्यवाद।

डॉ. गिरिजा व्यास, शिक्षा राज्य मंत्री, राजस्थान सरकार, का मुख्य अतिथि के रूप में अभिभाषण

परम श्रद्धेय शिक्षा मंत्री जी, साही साहब, आदरणीय डॉ. मण्डन मिश्र जी, परम श्रद्धेय अग्रवाल साहब, आदरणीय खरे साहब, बंसल जी, बाहर से पधारे हुए सभी गणित के मर्मज्ञ, वैदिक ज्ञान के मर्मज्ञ एवं सभी महानुभाव।

दरअसल कुछ दिन पूर्व ही जब संस्कृत विभाग का कार्य मुझे सौंपा, मैं पहले ही प्रोग्राम में गई और मैंने वहाँ यह आशा व्यक्त की कि संस्कृत में संस्कृत साहित्य और दर्शन के क्षेत्र में तो बहुत कुछ खोज भी हो चुकी है, लेकिन अभी गणित और ज्योतिष के क्षेत्र में बहुत कुछ करना है। और मुझे वहीं मण्डन मिश्र जी ने बताया कि इस विश्वविद्यालय में 25 तारीख से वैदिक गणित पर कार्यगोष्ठी हो रही है, मुझे बहुत खुशी हुई।

दरअसल 1979 में जब मैं अमेरिका में पढ़ा रही थी, तब डॉ. मैनन, यूनिवर्सिटी के वहाँ पर थे और तब शायद हमने अपना प्रथम आर्यभट्ट छोड़ा था, उसी दिन इण्डियन एसोसिएशन की तरफ से एक गोष्ठी थी, हमने खुशियाँ मनाई थीं। उन्होंने एक बात कही थी कि 21वीं सदी पर आप लोग दस्तक देने जा रहे हैं, नये विज्ञान के आप लोग जानकार हैं, लेकिन मुझे दुःख के साथ कहना पड़ रहा है कि अपने वैदिक ज्योतिष और वैदिक गणित को आप लोग भूल रहे हैं और उसी वर्ष डॉ. ए.जे. एयर अभी दुनिया के महानतम दार्शनिक हैं, शिकागो यूनिवर्सिटी आये और उन्होंने यह कहा कि "आप लोग 'लॉजिकल पोजिटिविज्म' और 'वैस्टर्न फिलॉसफी' के पीछे क्यों भागते हैं? आपके यहाँ तो दर्शन की सारी विधाएँ हैं, बल्कि मुझे भी कुछ दीजिये" और मुझे कहते हुए सुखद आश्चर्य है कि मैंने उनके "लोजिकल पोजिटिविज्म" के अनुरूप बौद्ध दर्शन के कुछ ग्रंथ उन्हें दिये और उसके ऊपर ही उनकी "रिसर्च" किताब छपी 1982 में। तब से मन में एक उत्कण्ठा थी कि वैदिक ग्रंथ के सन्दर्भ में और नव्यन्याय के संदर्भ में हम कुछ सोचें, ताकि विश्व को एक नई दिशा बोध दे सकें। भाग्यवश संस्कृत विभाग एक छोटे से हिस्से के रूप में, शिक्षा के एक छोटे से हिस्से के रूप में मेरे पास है। अलग विभाग होते हुए भी, और सबसे बड़ी बात डॉ. आर.पी. अग्रवाल जैसे वाइस चान्सलर यहाँ पर हैं, तो हम वैदिक गणित के सन्दर्भ में कुछ कर पायेंगे।

भारतीय समस्त ज्ञान-विज्ञान एवं कलाओं का स्रोत वैदिक वाङ्मय है। आपने अभी सबने स्वीकार किया। परा एवं अपरा नाम की दोनों विद्याएँ वैदिक वाङ्मय की अपूर्व निधि हैं। ज्योतिष, प्राचीन गणित एवं खगोल शास्त्र जिसे कहा जाता है, वेदपुरुष का नेत्र है। वैदिक श्रौत, श्राद्ध कर्मों के अनुष्ठान में सुदूर अंतरिक्ष में स्थित नक्षत्रों के लिए वैदिक गणित का अन्वेषण किया गया है। जिस आचार्य के नाम से जो-जो आविष्कार या अन्वेषण हुआ, उसी खोज या कृति को उन्हीं के सिद्धान्त के रूप में अभेद किया गया। जैसे ब्रह्म द्वारा ब्रह्म-सिद्धान्त, सूर्य द्वारा सूर्य-सिद्धान्त, वशिष्ठ-सिद्धान्त या शाकल्य-संहिता आदि। इसी प्रकार प्राचीन ऋषियों ने शुल्ब सूत्रों द्वारा वैदिक गणित का प्रतिपादन किया, जिनमें प्रमुख हैं कात्यायन-शुल्ब-सूत्र और बोधायन-शुल्ब-सूत्र आदि। इन सबका रचना-काल, मैं तो कहूँगी, कम से कम 8000 वर्ष पूर्व माना जाना चाहिए। क्योंकि ये सब पाणिनि और महाभारत काल के पूर्व के हैं। महाभारतोत्तर काल के खगोल वेत्ताओं में ब्रह्मशूल्य, पाराशर, शाकल्य आदि ऋषियों और आर्यभट्ट, ब्रह्मगुप्त श्रीधर, कोणार्क, भास्कराचार्य, मिहिर, कमलाकर, महीषवर बहुत ही आचार्यों ने गणित के विकास को अपनी कृतियों द्वारा दिग्दर्शन कराया। ज्योतिषशास्त्र-वेत्ताओं के अनेक उदाहरण हमें मिलते हैं, जैसे मैं खगोल विषय में आचार्य भास्कर का उदाहरण देना चाहती हूँ।

"कोटिघ्नैर्नखनन्दषट्क-नख-भू-भूभृद्-भुजङेन्दुभिः।
ज्योतिःशास्त्रविदो वदन्ति नभसः कक्षामिमां योजनैः।।"

अर्थात् आकाश के 18 पद्म, 71 नील, 20 खरब, 69 अरब, 20 करोड़ योजन इस आकाश की कक्षा का मान है। यहाँ आपसे मैं यह निवेदन करना चाहती हूँ कि कुछ हमारे वैज्ञानिक और गणितज्ञ इन पुराने, जो योजना आदि दी हुई है, उसी पर अडिग हैं। मुझे शिकागो यूनिवर्सिटी में डॉ. कृष्णामूर्ति के दर्शन हुए, जिनका विवाद वहाँ के दूसरे गणित शास्त्रिओं, ज्योतिष शास्त्रिओं के साथ इसलिए चल रहा था कि वह पुरानी योजना है, उस पर अड़े हुए थे। मैं यहाँ पर यह भी निवेदन करना चाहती हूँ कि पुरानी पद्धतियाँ तो थीं ही, नवीन पद्धति के साथ जोड़ने का भी प्रयास किया जाना चाहिए। क्योंकि आकाश के जैसे कक्षा में तत्तद् ग्रहों में मगण का भाग देने से कक्षा आती है, इस सन्दर्भ में, मैं अधिक तो नहीं कहना चाहती, क्योंकि अग्रवाल साहब और खरे साहब जैसे लोग हैं। लेकिन शायद पुरानी इस पद्धति को अब बदल दिया गया है। मैं हमारे पुराने वैदिक शास्त्रियों से यह भी निवेदन करना चाहती हूँ कि आकाश की कक्षा को गिनने के संबंध में जो नई खोज हुई है, उसे भी अपनी खोज के साथ जोड़ने का प्रयास करें। प्राचीन काल में योजन, कोश, चटुष्ड्वात्मक परिणाम या उसी प्रकार खगोलशास्त्र के क्षेत्र में बहुत कार्य किया गया। इसी प्रकार जो वर्तमान में व्यास, परिधि और क्षेत्रफल का प्रकार बताया गया है, वह भी

भास्कराचार्य ने "व्यासे भनन्दाग्निहते विभक्ते खबाणसूर्यः परिधिः व सूक्ष्मः। दाविंशतिघ्ने विहृतेऽथ शैलैः स्थूला अपि वा स्याद् व्यवहारयोग्यः" के अनुरूप है। अर्थात् व्यास को 3927 से गुणा करके 1250 का भाग देने से जो प्राप्त हो, वह सूक्ष्म परिधि का मान होगा। इसी प्रकार बीजगणित भास्कराचार्य के वर्गान्तर योगान्तर घात सम सूत्र से आज भी प्रभावित है। बीजगणित के कुट्टक अनेक वर्ग एवं समीकरण एवं एकवरण मध्यमा हरण गणित के आचार्य पद्मनाभ, श्रीधर गुप्त से प्रस्फुटित होकर आज भी विद्यमान हैं।

इसी प्रकार पाटिगणित के रूपस्थान विभाग और अंत गुणंत की प्रक्रियाएँ क्रमशः वर्तमान, गुणा, लघुविरुद और गुणन खंड की प्रक्रियाओं के अनुरूप है। व्यवहार गणित आज भी मौजूद है और उसकी सरल प्रक्रिया इसी प्रकार क्षेत्र व्यवहार में भुजकमोटिकण में से यानि दो अथवा दो के योग को जानकर तीनों के अलग-अलग जानने के प्रकार क्षेत्रों की आभा का आधार लम्ब ज्ञान आदि से संबंधित अनेक प्रश्न प्राचीन गणित के ज्ञान कोष्ठों की प्रतिष्ठा बढ़ा रहा है। मैं तो आज के दिन यही कहना चाहती हूँ कि वेद-विद्या विज्ञान के कात्यायन-शुल्ब-सूत्र, बौधायन-सूत्र, आचार्य ब्रह्म का ब्रह्म-सिद्धान्त, सूर्य-सिद्धान्त और भास्कराचार्य के सिद्धान्त-शिरोमणि में प्रारंभिक गणित से लेकर खगोल प्रयत्न गणित के विभिन्न आयामों का वर्णन जो हमें प्राप्त हो रहा है, उस पर पुनः खोज की जानी चाहिए।

सिद्धान्त-तत्त्व-विवेककार आचार्य कमलाकर के अनेक ग्रंथों में सूक्ष्मजा, कोटिजा, उत्क्रमित्जा साधन करके उनके द्वारा पम्प्रतिसा, समत्रिभुज आदि कुण्डों के निर्माण की प्रक्रिया और वराहमिहिराचार्य ने अपनी संहिता में जो वास्तु-प्रकरण के विभिन्न प्रकारों, भावनाओं, राजमार्गों तथा नगर निर्माण कला का जो वर्णन किया है, जिसके आधार पर जयपुर नगर का भी निर्माण किया गया है, उस पर भी खोज होनी आवश्यक है। मैं एक बात और निवेदन करना चाहती हूँ वेद गणित और वेद के जो हमारे विज्ञान के अंग हैं, वे इसलिए छूट गये कि कालान्तर में हमने धर्म को दर्शन के साथ जोड़ दिया। हमारे यहाँ पर यह तो नहीं था कि We cannot serve two masters together, विज्ञान और ज्ञान, परा विद्या और अपरा विद्या। लेकिन इसका अर्थ यहाँ यह भी कदापि नहीं था कि धर्म के साथ समवेत होकर के, जैसा कि मण्डन मिश्र जी कह रहे थे—हमने मीमांसा के मूल अर्थ को खो दिया। आज आवश्यकता इस बात की है कि हम धर्म से कम से कम विज्ञान को, गणित और दर्शन को अलग करके ऐसी खोज कर सकें, जिससे कि 21वीं सदी पर दस्तक देते सम्पूर्ण विश्व को हम नये आयाम दे सकें। मुझे बहुत प्रसन्नता है और मैं डॉ. अग्रवाल जी की विशेष रूप से आभारी हूँ कि उन्होंने एक ऐसे भूले हुए विषय पर संगोष्ठी की, जो विश्व को नये आयाम ही नहीं प्रदान कर सकेगी, बल्कि विश्व को चमत्कृत करने का जिसके पास साधन होगा। मुझे आज के

दिन यहाँ बुलाया, उसके लिए मैं बहुत धन्यवाद ज्ञापित करना चाहती हूँ। मैं विशेष रूप से डॉ. साही साहब की अनुगृहीत हूँ, जो पार्लियामेंट चलते हुए भी यहाँ पर तशरीफ लाये हैं। मुझे लगता है कि तीन-चार दिन की गोष्ठी में आप लोग नये निर्णयों पर पहुँचेंगे और वैदिक गणित के नये आयामों को प्रस्तुत कर सकेंगे। जय हिन्द। धन्यवाद।

विशिष्टं भाषणम्

डॉ. मण्डनमिश्रः

संमाननीयाः केन्द्रीय-शिक्षा-संस्कृतिराज्यमंत्रिणः श्रीललितेश्वरप्रसादशाहीमहोदयाः, राजस्थानस्य शिक्षा-संस्कृतिराज्यमंत्रिवर्या माननीया डॉ. गिरिजाव्यासमहाशयाः, राजस्थान-विश्वविद्यालयस्य कुलपतयो मनीषिवरेण्याः डॉ. रत्नप्रकाश-अग्रवालमहाभागाः, कार्यशालाया निदेशकाः श्रीहरीशखरेमहानुभावाः, उपस्थिता विद्वांसो, भ्रातरो, भगिन्यश्च।

गुणगरिम्णा संपन्नस्य अस्य राजस्थानविश्वविद्यालयस्य प्राङ्गणे राष्ट्रिय-वेद-विद्या-प्रतिष्ठान-भारतीय-दार्शनिकअनुसंधानपरिषत्-राष्ट्रियसंस्कृतसंस्थान-राजस्थान-विश्वविद्यालयानां संयुक्ततत्त्वावधाने समायोजितायामस्यां कार्यशालायां वैदिकगणितमधिकृत्य शास्त्रचर्चां विधातुं वयं सर्वेऽपि समुपस्थिताः। अयं हि राजस्थानप्रदेशः एकत्र शौर्ये, अपरत्र भारतीय-संस्कृतेः भारतीयमर्यादानां, भारतीयविद्यानां च संरक्षणाय सदैव जागरूकः समासीत्। अस्य विभिन्नानां प्राचीनानां राज्यानां गौरवपूर्णमितिवृत्तम् एतेषु क्षेत्रेषु अस्य प्रदेशस्य विशिष्टं योगदानं स्मारयति। यस्मिन् हि नगरे इयं चर्चा प्रवर्तते, तत् जयपुरं नाम नगरम् एकत्र कलायां, सौन्दर्ये विविधासु विद्यासु च विशिष्टं स्थानं धारयति। इदं च संस्कृतक्षेत्रे कमप्यनुपमं महिमानं बिभर्ति। अस्य हि गणना अपरकाशीत्वेन इतिहासकारैः कृता। अत्रत्यानां महाराजानामियमेव कामनाऽऽसीत्—

"वाराणसेयतु सदा जयपत्तनं मे"।

अत एव विविधेषु विषयेषु वाचस्पतिसमा मनीषिणो नगरमेतदलंचक्रुः। अत्र हि वेद-विद्याया महान् प्रचारः प्रसारश्च आसीत्। अस्य राज्यस्य शासकैः देशस्य विभिन्नेभ्यो आन्ध्र-बिहारमहाराष्ट्रगुर्जरादिप्रदेशेभ्यो विशिष्टा विद्वांसः समामंत्रिताः सम्मानिताश्च राजकीय-सम्मानेन पर्याप्तया संपदा च। केवलं सार्धशतकात् प्रागेव इयं नगरी विद्यावाचस्पतिभिः श्रीमधुसूदनओझामहोदयैः समलंकृता, यैः सर्वथा इदम्प्रथमतया प्रवर्तितं वैदिकं विज्ञानं परमोत्कृष्टज्ञानधारात्वेन प्रतिष्ठितम्।

इदमासीत् चमत्कृतिपूर्णं विवेचनं यत् श्रीओझामहोदयैः सर्वतः पूर्वं विश्वस्य समक्षं प्रस्तुतम्। तैः प्रवर्तितस्यास्य वैदिकविज्ञानस्य महामहोपाध्यायश्रीगिरिधरशर्मचतुर्वेदैः, श्रीमोतीलालशास्त्रिभिः, श्रीवासुदेवशरणअग्रवाल-प्रभृतिभिश्च पण्डितप्रवरैः सुमहद् व्याख्यानं कृतम्। तेषां प्रियतमेन शिष्येण स्वनामधन्येन श्रीमोतीलालशास्त्रिणा केवलं स्वकीयैः हस्ताक्षरैः लक्षपरिमितानि पृष्ठानि विषयमिममधिकृत्य लिखितानि, येषु, प्रायशः

पञ्चाशत् सहस्राणि अप्रकाशितानि वर्तन्ते। महानयं सन्तोषस्य विषयो यत् श्रीशास्त्रि-परिवारस्य प्रयत्नेन तदिदं साहित्यं दुर्गापुरस्थिते तैरेव सुप्रतिष्ठापिते मानवाश्रमे संरक्षितं वर्तते, तस्य च विश्वस्मिन् प्रचाराय राजस्थानपत्रिकायाः संस्थापकाः संपादकाः श्रीकपूरचन्द्रकुलिश-महोदयाः सततं प्रयतमाना वर्तन्ते, तैर्महता योगदानेन प्रकाशिता अनेके भागाः, प्रसारितानि विशिष्टानि भाषणानि विभिन्नेषु राष्ट्रेषु। राजस्थानपत्रिकासदृशेन महत्तमेन दैनिकपत्र-माध्यमेन च लेखैः संपादकीयैश्च सुप्रचारितः पुनरुज्जीवितश्चायं विषय इति महतो हर्षप्रकर्षस्य विषयः।

अतो नूनमिदं क्षेत्रं वैदिकविज्ञानक्षेत्रम्, अत्र हि महाराजेन श्रीजयसिंहेन संस्थापिता वेधशाला नक्षत्रज्ञानं सर्वथा प्रत्यक्षं कारयति। अतः सर्वथा समुपयुक्तोऽयं प्रदेशः, वैदिकगणितकार्यशालायाः कृते। उर्वरा चेयं भूमिः फलप्राप्तये। मङ्गलमयेऽस्मिन् प्रसङ्गे वयं सर्वेऽपि राजस्थानवासिनः स्वकीयं भागधेयमहिमानं प्रशंसामहे, तत्रभवतां दर्शनेन चात्मानं कृतकृत्यतमं मन्यामहे।

गणितशास्त्रं हि मूलं समस्तस्यापि ज्ञानविज्ञानस्य। अस्मिन् हि क्षेत्रे भारतीयैः मनीषिभिः समुल्लेखनीयानि योगदानानि कृतानि। तेषां गणनाऽपि सुदुष्करा। तेष्ववदानेषु तत्रभवतां दिव्यात्मनां जगद्गुरुणां पुरीशंकराचार्याणां श्रीभारतीकृष्णतीर्थमहोदयानां वैदिकं गणितं विशेषतत्त्वज्ञानक्षेत्रेऽस्माकं देशस्य नवीनतमं योगदानम्। गते शतके संसारस्य विभिन्नेषु देशेषु स्वकीयानि सूत्राणि तैः व्याख्यातानि तेषां च महत् स्वागतं सर्वत्राभवत्। तदनुसारं संप्रत्यपि वैदिकगणितविषये चर्चाः कक्षाश्च प्रवर्तन्ते, मनीषिणां चेदं मतं यदिदं गणितं सर्वथा विस्मयास्पदं, सरलं, सुगममत एव विकासाय विश्वस्य कल्याणाय चात्यन्तमुपयुज्यते। गतेषु संवत्सरेषु पूज्यवरेण्यानां श्रीभारतीकृष्णतीर्थमहानुभावानामनुयायिभिः डॉ. नरेन्द्रपुरी, डॉ. एस.एन. अग्रवालः, प्रो. श्रीकराडे, श्रीमती मञ्जुला त्रिवेदी-प्रभृतिभिराचार्यैः विषयेऽस्मिन् बहु परिश्रान्तम्। श्रीईश्वरभाईपटेलसदृशैश्च शिक्षाशास्त्रिभिः महर्षिवैदिकविज्ञानअकादमी-सभापतित्वेन लोकशिक्षायै एतस्य प्रयोगाः कृताः। त इमे प्रयोगा यथाधुनिकतमैर्गणित-विद्याविशारदैरनुमन्येरन् पाठ्यक्रमेषु समाविष्टाश्च स्युरित्यर्थं लोकसभायां, समाचारपत्रेषु च निवेदनानि कृतानि प्रतिदिनं च क्रियन्ते। महाकविना कालिदासेन कथितम्—"आ परितोषाद्विदुषां न साधु मन्ये प्रयोगविज्ञानम्"। अत एतेषां प्रयोगानां साफल्ये गणिताचार्याणां मान्यताप्राप्तये अयं समारम्भः।

इयं हि कार्यशाला वस्तुतो मानवसंसाधनविकासमन्त्रिणां संमाननीया श्री पी.वी. नरसिंह-रावमहोदयानां निर्देशेनात्र प्रवर्तते। संसत्सदस्यैः विचारकमूर्धन्यैः डॉ. ए.के. पटेलमहोदयैः प्रश्नमाध्यमेन न केवलं शासनस्य, संसदः, अपि तु अखिलस्यापि जगतोऽवधानं समाकृष्टम्। तस्य प्रश्नस्य ऐतिहासिके समुत्तरे श्रीपी.वी. नरसिंहरावमहानुभावैः वैदिकगणितस्य

कतिपयानां सूत्राणां परमं विद्वत्तापूर्णं विश्लेषणं प्रश्नोत्तरप्रसंगे श्रीललितेश्वरप्रसादशाही-महोदयैः, वैदिकगणितविषये संसारस्य विभिन्नेषु राष्ट्रेषु प्रचालितानां कार्यक्रमाणां विवरणं च कृतम्। विषयोऽयं सुप्रकाशितः, समाचारपत्रैश्च महता समादरेण सुप्रचारितः। एतत् सर्वं प्रकरणं कार्यशालाया अस्याः प्रेरणास्रोतः, सौम्यमूर्तयो विद्याविनयविवेकावताराः विशेषशिक्षासचिवाः श्रीकिरीटजोशीमहोदयाश्च अस्याः सूत्रधारत्वेन विराजन्ते इति स्वर्णसुरभियोगः प्रकल्पितः।

विशेषतः अत्रभवन्तः महामहिमशालिनः श्रीललितेश्वरप्रसादशाहीमहाशयाः अस्मान् अनुग्रहीतुं समुपस्थिता इति गौरवस्यायं विषयः। श्रद्धेयाः श्रीशाहीमहोदयाः बिहार-प्रदेशस्यालंकारभूताः बिहारराज्यस्य तकनीकीशिक्षायाः, स्वास्थ्यशिक्षायाश्च प्रकल्पकाः, संस्थापका अनेकेषां महाविद्यालयानां, बिहारराज्यस्य कृषिस्वास्थ्यादिविभिन्नविभागानां मन्त्रित्वेन विकासस्य कर्णधाराः, अत एव शिक्षायाः संस्कृतेश्च संरक्षकत्वेन प्रसिद्धाः, सम्प्रति केन्द्रशासने शिक्षासंस्कृतिराज्यमन्त्रिपदे प्रतिष्ठिता इति सौभाग्यं राष्ट्रस्य। अस्याः कार्यशालायाः समुद्घाटनाय एतेषामत्र पदार्पणं तेषां भारतीयविद्यासु महतीमभिरुचिं प्रकटयति। एतेषां समागमनं संसूचयति केन्द्रशासनस्य सहयोगं समथर्नं च। मन्ये एभिः प्रज्वलितोऽयं ज्ञानप्रदीपः वैदिकगणितस्य ज्योतिः सर्वत्र प्रसारयिष्यति, इत्यत्र नास्ति संशयलेशः। इमां हि कार्यशालां संमानितातिथित्वेन संबोधयितुं राजस्थानस्य संस्कृत-शिक्षामंत्रिपदमलंकुर्वाणा डॉ. गिरिजाव्यासमहोदयाः समायाता इति विशेषेण संशोभितेयं कार्यशाला। एता हि दर्शनशास्त्रस्य विदुष्यः शिक्षायाश्च मूर्तरूपत्वेन सर्वत्र समादृताः। अत्रभवतीषु शासकत्वं दार्शनिकत्वञ्चैकत्र संगतमिति सौभाग्यस्थानं सर्वस्यापि राजस्थान-प्रदेशस्य कृते। वस्तुतो वयं सर्वेऽप्युपकृता एतासां सहयोगेन।

सेयं कार्यशाला राजस्थानविश्वविद्यालये समायोजिता। विश्वविद्यालयोऽयं सुप्रतिष्ठितः गौरवशालितया राराजते। अस्य कुलपतयः श्रीरत्नप्रकाशअग्रवालमहोदया विश्वप्रसिद्धा गणितशास्त्रविद्वांसः, कुशलाः प्रशासकाश्च। तैरस्याः कार्यशालायाः स्वागतव्यवस्था कृतेति वयमधमर्णास्तेषाम्। अस्याश्च निदेशकत्वं गणितशास्त्रस्य भारतीयपाश्चात्त्यपक्षयोः निष्णातैः जगति प्रख्यातैः मनीषिभिः श्रीहरीशखरेमहानुभावैः स्वीकृतमिदं सर्वमस्माकं साफल्यस्य आधारशिलात्वेनोपकल्पते। एतेषामाधुनिकविशारदानां विदुषां नेतृत्वे, भवादृशानां समीक्षकाणां वैदिकगणितज्ञानम् आधुनिकगणनाविशारदानां संस्कृतविदुषां च समागमोऽयं त्रिवेणीत्वेन संसारस्य पथप्रदर्शनाय भविष्यतीति निश्चप्रचम्।

अहं पुनः राष्ट्रियवेदविद्याप्रतिष्ठान-भारतीयदार्शनिकअनुसंधानपरिषद्-राष्ट्रियसंस्कृत-संस्थान-राजस्थानविश्वविद्यालयपक्षतः समेषामपि तत्रभवतां स्वागतमभिनन्दनं च व्याहरामि। विशेषतश्च माननीयानां श्रीललितेश्वरप्रसादशाहीमहोदयानां मनीषिवरेण्यानां डॉ. गिरिजाव्यासमहानुभावानां च कृते महतीं कृतज्ञतां ज्ञापयन् विरमामि।

Vedic Mathematics
The Deceptive Title of Swamiji's Book

K.S. SHUKLA

The title of the book, *Vedic Mathematics or Sixteen Simple Mathematical Formulae from the Vedas,* written by Jagadguru Svāmī Śrī Bhāratī Kṛṣṇa Tīrthaji Mahārāja, Śaṅkarācārya of Govardhana Matha, Puri, bears the impression that it deals with the mathematics contained in the Vedas — *Ṛgveda, Sāmaveda, Yajurveda* and *Atharvaveda.* This indeed is not the case, as the book deals not with Vedic Mathematics but with modern elementary mathematics up to the Intermediate standard. In his Foreword to Swamiji's book, V. S. Agrawala, the editor, writes:

> "The question naturally arises as to whether the sūtras which form the basis of this treatise exist anywhere in the Vedic literature as known to us. But this criticism loses all its force if we inform ourselves of the definition of Veda given by Sri Sankaracharya himself as quoted below:
>
>> 'The very word "Veda" has this derivational meaning, i.e. the fountain-head and illimitable store-house of all knowledge. This derivation, in effect, means, connotes, and implies that the Vedas *should* contain within themselves all the knowledge needed by mankind relating not only to the so called "spiritual" (or other-worldly) matters but also to those usually described as purely "secular", "temporal", or "worldly" and also to the means required by humanity as such for the achievement of all-round, complete and perfect success in all conceivable directions and that there can be no adjectival or restrictive epithet calculated (or tending) to limit that knowledge down in any sphere, any direction or any respect whatsoever.
>>
>> 'In other words, it connotes and implies that our ancient Indian Vedic lore should be all-round, complete and perfect and able to throw the fullest necessary light on all matters which any aspiring seeker after knowledge can possibly seek to be enlightened on.'

"It is the whole essence of his assessment of Vedic tradition that it is not to be approached from a factual stand-point but from an ideal stand-point, viz. as the Vedas as traditionally accepted in India as the repository of all knowledge *should be* and not what they are in human possession. That approach entirely turns the tables on all critics, for the authorship of Vedic mathematics then need not be laboriously searched in the texts as preserved from antiquity."

In his preface to his *Vedic Mathematics*, Swamiji has stated that the sixteen *sūtras* dealt with by him in that book were contained in the *Pariśiṣṭa* (the Appendix) of the *Atharvaveda*. But this is also not a fact;[1] for they are untraceable in the known *Pariśiṣṭas of the Atharvaveda* edited by G.M. Bolling and J. von Negelein (Leipzig, 1909-10). Some time in the 1950 when Swamiji visited Lucknow to give a black-board demonstration of the sixteen *sūtras* of his 'Vedic Mathematics' at the Lucknow University, I personally went to him at his place of stay with Bolling and Negelein's edition of the *Pariśiṣṭas of the Atharvaveda* and requested him to point out the places where the sixteen *sūtras* demonstrated by him occurred in the *Pariśiṣṭas*. He replied off hand, without even touching the book, that the sixteen *sūtras* demonstrated by him were not in those *Pariśiṣṭas*; they occurred in his own *Pariśiṣṭa* and not in any other.

As regards the *Pariśiṣṭas* of the *Atharvaveda* referred to by Swamiji, V.S. Agrawala says:

"The Vedas are well-known as four in number, Ṛk, Yajus, Sāma and Atharva, but they have also the four Upavedas and the six Vedāṅgas all of which form an individual corpus of divine knowledge as it once was and as it may be revealed. The four Upavedas (associated with the four Vedas) are as follows:

Vedas	*Upavedas*
Ṛgveda	Āyurveda
Sāmaveda	Gandharvaveda
Yajurveda	Dhanurveda
Atharvaveda	Sthāpatyaveda

[1]In order to put the matter in proper perspective the views of the Jagadguru Śaṅkarācārya contained in the chapter on Vedic mathematics in his book *Vedic Metaphysics* have been given at the end of this publication as Appendix II. (Editorial note).

"In this list the Upaveda or Sthāpatya or engineering comprises all kinds of architectural and structural human endeavour and all visual arts. Swamiji naturally regarded mathematics or the science of calculations and computations to fall under this category."

"In the light of the above definition and approach must be understood the author's statement that the sixteen *sūtras* on which the present volume is based form part of a Pariśiṣṭa of the Atharvaveda. We are aware that each Veda has its subsidiary apocryphal texts some of which remain in manuscripts and others have been printed but that formulation has not closed. For example, some Pariśiṣṭas of the Atharvaveda were edited by G.M. Bolling and J. von Negelein, Leipzig, 1909-10. But this work of Sri Sankaracharyaji deserves to be regarded as a new Pariśiṣṭa by itself and it is not surprising that the sūtras mentioned herein do not appear in the hitherto known Pariśiṣṭas."

V.S. Agrawala's verdict that the work of Śrī Śaṅkarācārya deserves to be regarded as a new pariśiṣṭa by itself is fallacious. The question is whether any book written in modern times on a modern subject can be regarded as a Pariśiṣṭa of a Veda. The answer is definitely in the negative.

From what has been said above it is evident that the sixteen *sūtras* of Swamiji's *Vedic Mathematics* are his own compositions, and have nothing to do with the mathematics of the Vedic period. Although there is nothing Vedic in his book, Swamiji designates his Preface to the book as 'A Descriptive Prefactory Note on the Astounding Wonders of Ancient Indian Mathematics' and at places calls his mathematical processes as Vedic processes.

The deceptive title of Swamiji's book and the attribution of the sixteen *sūtras* to the *Pariśiṣṭas* of the *Atharvaveda*, etc., have confused and baffled the readers who have failed to recognize the real nature of the book, whether it is Vedic or non-Vedic. Some scholars, in their letters addressed to me, have sought to know whether the sixteen *sūtras* stated by Swamiji occurred anywhere in the Vedas or the Vedic literature.

Even the Rashtriya Veda Vidya Pratishthan, under whose auspices this Workshop on Vedic Mathematics has been organized, in their circular letter issued through the Ministry of Human Resource Development, are under the impression that the sixteen *sūtras* were actually reconstructed from materials in the various parts of the Vedas and the sixteen formulae contained in them were based on an Appendix of the *Atharvaveda*, which Appendix was not known to exist before the publication of Swamiji's book.

Let us now examine briefly the contents of that part of Swamiji's book which demonstrates the sixteen *sūtras*. These are divided into 40 chapters which run as follows :

Ch.1. deals with the conversion of vulgar fractions into decimal or recurring decimal fractions. Here it may be remarked that nobody in the Vedic period could think of decimal or recurring decimal fractions. The decimal fractions were first introduced by the Belgian mathematician Simon Stevin in his book *La Disme* which was published in A.D. 1585. The decimal point (.) was used for the first time by Lemoch of Lemberg. The recurring decimal point ($.\dot{6}$ for .666...) is the invention of Nicholas Pikes (A.D. 1788).

Chs.2 and 3 deal with methods of multiplication and chs. 4 to 6 and 27 with methods of division. All these methods are quite different from the traditional Hindu methods.

Chs.7 to 9 deal with factorization of algebraic expressions, a topic which was never included in any work on Hindu algebra.

Ch.10 deals with the H.C.F. of algebraic expressions. This topic also does not find place in Hindu works on algebra.

Chs.11 to 14 and 16 deal with the various kinds of simple equations. These are similar to those occurring in modern works on algebra.

Chs.1 and 20-21 deal with the various types of simultaneous algebraic equations. These are also similar to those taught to Intermediate students and do not occur in ancient Hindu works on algebra.

Ch.17 deals with quadratic equations; ch. 18 with cubic equations; and ch. 19 with biquadratic equations.

Ch.22 deals with successive differentiation, covering the theorems of Leibnitz, Maclaurin and Taylor, among others; ch. 23 with partial fractions; and ch. 24 with integration by partial fractions. These are all modern topics.

Ch.25 deals with the so called *Kaṭapayādi* system of expressing numbers by means of letters of the Sanskrit alphabet. It is called by Swamiji

by the name 'the Vedic numerical code' although it has not been used anywhere in the Vedic literature.

Ch.26 deals with the recurring decimals; ch.28 with the so-called auxiliary fractions; and chs. 29 and 30 with divisibility and the so-called osculators. These topics too do not find place in the Hindu works on algebra.

Ch.31 deals with the sum and difference of squares.

Chs.32 to 36 deal with squaring and cubing, square-root and cube-root.

Ch.37 deals with Pythagoras Theorem and ch.38 with Appllonius Theorem.

Ch.39 deals with analytical conics, and finally ch.40 with miscellaneous methods.

From the contents it is evident that the mathematics dealt with in the book is far removed from that of the Vedic period. Instead, it is that mathematics which is taught at present to High School and Intermediate classes. It is indeed the result of Swamiji's own experience as a teacher of mathematics in his early life. Not a single method described is Vedic, but the Swamiji has declared all the methods and processes explained by him as Vedic and ancient.

Let us now say a few words regarding the mathematics which was known in the Vedic period, i.e. during the period ranging from *c.* 2500 B.C. to *c.* 500 B.C.

Works of this period dealing exclusively with mathematics have not survived the ravages of time and our knowledge regarding mathematics of this period is based on the religious works of this period, viz. the Vedic Saṃhitās, the Brāhmaṇas and the Vedāṅgas. The religious works of the Buddhists and the Jainas and the *Āryabhaṭīya* of Āryabhaṭa (A.D. 499) give some idea of the development of mathematics from 500 B.C. to A.D. 500.

A study of the Vedic works reveals that by 500 B.C the Hindus were well-versed in the use of numbers. They knew all the fundamental operations of arithmetic, viz. addition, subtraction, multiplication, division, squaring, cubing, square-root and cube-root. They were also well-versed in the use of fractional numbers and surds, mensuration and

construction of simple geometrical figures, and could solve some algebraic problems also.

In arithmetic, they were masters of numbers and could use large numbers. They had developed an extremely scientific numeral terminology based on the scale of 10. In the *Yajurveda-Saṃhitā* (*Vājasaneyi*, XVII.2) we have the following list of numeral denominations proceeding in the ratio of 10:

> *eka* (1), *daśa* (10), *śata* (100), *sahasra* (1000), *ayuta* (10000), *niyuta* (10^5), *prayuta* (10^6), *arbuda* (10^7), *nyarbuda* (10^8), *samudra* (10^9), *madhya* (10^{10}), *anta* (10^{11}), and *parārdha* (10^{12}).

The same list occurs in the *Taittirīya-Saṃhitā* (IV. 40.11.4 and VII.2.20.1), and with some alterations in the *Maitrāyaṇī* (II.8.14) and *Kāṭhaka* (XVII. 10) *Saṃhitās* and other places.

The numbers were classified into even (*yugma*, literally meaning 'pair') and odd (*ayugma*, literally meaning 'not pair'). In two hymns of the *Atharvaveda* (XIX.22, 23), there seems to be a reference to the zero, as well as to the recognition of the negative number. The zero has been called *kṣudra* (trifling). The negative number is indicated by the term *anṛca*, while the positive number by *ṛca*.

The Vedic Hindus seem to have been interested in series or progressions of numbers as well. The following series are found to occur in the *Taittirīya-Saṃhitā* (VII. 2.12.17) :

1,3,5, ..., 19 ..., 29 ..., 39 ..., 99
2,4,6, ..., 20
4,8,12, ..., 20
5,10,15 ..., 100
10,20,30 ...,100

The arithmetic series were classified into even (*yugma*) and odd (*ayugma*) series. The following examples of these two categories are given in the *Vājasaneyi-Saṃhitā* (XVIII. 24, 25):

4, 8, 12, ..., 48
1, 3, 5, ..., 33

Of these two series, the second one is found to occur also in the *Taittirīya-Saṃhitā* (IV. 3.10). In the *Pañcaviṃśa-Brāhmaṇa* (XVIII. 3) is given a list

of sacrificial gifts which form the following series in geometrical progression:

24, 48, 96, 192, ... , 49152, 98304, 196608, 393216.

This series occurs also in the *Śrauta-sūtras.*

Some method for summing a series was also known. In the *Śatapatha-Brāhmaṇa* (X. 5.4.7), the sum of the series

$$3 \times (24 + 28 + 32 + \; + \ldots \text{ to 7 terms})$$

is stated correctly as 756. And in the *Bṛhaddevatā* (III.13) the sum of the series

$$2 + 3 + 4 + \ldots + 1000$$

is stated correctly as 500499.

From the method indicated by Baudhāyana for the enlargement of a square by successive addition of gnomons, it seems that the following result was known to him:

$$1 + 3 + 5 + \ldots + (2n + 1) = n + 1.$$

From the following results occurring in the *Śulba-sūtras* we find that the Vedic Hindus knew how to perform fundamental operations with fractional numbers:

$$7\tfrac{1}{2} \div \tfrac{1}{25} = 187\tfrac{1}{2}$$

$$\left(2\tfrac{1}{7}\right)^2 + \left(\tfrac{1}{2} + \tfrac{1}{12}\right)\left(1 - \tfrac{1}{3}\right) = 7\tfrac{1}{2}$$

$$7\tfrac{1}{9} = 2\tfrac{2}{3}$$

$$7\tfrac{1}{2} \div \tfrac{1}{15} \text{ of } \tfrac{1}{2} = 225$$

In geometry, the Vedic Hindus solved propositions about the construction of various rectilinear figures, combination, transformation and application of area, mensuration of areas and volumes, squaring of the circle and vice versa, and about similar figures. One theorem which was of the greatest importance to them on account of its various applications was the so-called

Pythagoras Theorem. It has been enunciated by Baudhāyana (800 B. C.) thus: 'The diagonal of a rectangle produces both (area) which its length and breadth produce separately' (*Baudhāyana-Śulba*, 1.48). That is, the square described on the diagonal of a rectangle has an area equal to the sum of the areas of the squares described on its two sides.

The converse theorem, viz. 'If a triangle is such that the square on one side of it is equal to the sum of the squares on the other two sides, then the angle contained by these two sides is a right angle', is not found to have been expressly stated by any Vedic geometrician. But its truth has been tacitly assumed by all of them and it has been most freely employed for the construction of a right angle.

In the course of construction of fire-altars, it was necessary to add together two or more figures such as squares, rectangles, triangles, etc., or subtract one of them from another. In the case of combinations of squares, mere application, repeated when necessary, of the Pythagoras Theorem was sufficient to get the desired result. But in the case of other figures, they had first to be transformed into squares before the theorem could be applied and the combined square was then used to be transformed into any desired shape.

The Vedic Hindus knew elementary treatment of surds. They were aware of the irrationality of $\sqrt{2}$ and attained a very remarkable degree of accuracy in calculating its approximate value, viz.

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3.4} - \frac{1}{3.4.34}, \text{nearly.}$$

In terms of decimal fraction this works out to $\sqrt{2} = 1.4142156\ldots$ According to modern calculation $\sqrt{2} = 1.414213\ldots$, so that the Hindu approximation is correct up to the fifth place of decimals, the sixth place being too large.

There have been many speculations as to how the value of $\sqrt{2}$ was determined in that early time to such a high degree of approximation. The Kerala mathematician Nīlakaṇṭha (A.D. 1500) was of the opinion that Baudhāyana assumed each side of a square to consist of 12 units. Thus the square of its diagonal was equal to 2.12^2. Now

$$2.12^2 = 288 = 289 - 1 = 17^2 - 1$$

$$\therefore 12.\sqrt{2} = \sqrt{17^2 - 1}$$

$$= 17 - \frac{2}{2.17}, \text{ nearly}$$

$$\text{Hence } \sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3.4} - \frac{1}{3.4.34}, \text{ nearly}$$

The same hypothesis has been suggested by G.Thibaut (*Śulba-sūtras*, pp. 13 ff.). Several other methods have been given by B.Datta and others.

Another similar result is

$$\sqrt{3} = 1 + \frac{2}{3} + \frac{1}{3.5} - \frac{1}{3.5.52}, \text{ nearly,}$$

whose derivation, as suggested by Nīlakaṇṭha, is as follows:

$$\begin{aligned}\sqrt{3} &= \frac{1}{15}\sqrt{3.\,15^2} \\ &= \frac{1}{15}\sqrt{26^2 - 1} \\ &= \frac{1}{15}\left(26 - \frac{1}{2.26}\right), \text{ nearly,} \\ &= 1 + \frac{2}{3} + \frac{1}{3.5} - \frac{1}{3.5.52}, \text{ nearly.}\end{aligned}$$

An Overview of Vedic Mathematics

NARENDRA PURI

The Vedas are the source of all knowledge and of all sciences. They are not only the oldest but also the best. The Vedas have guided millions of aspirants on the path of knowledge. With the passage of time most of the Vedic knowledge has been scattered. Vedic Mathematics is no exception to it. It is inspiring to observe that deep interest is being taken the world over in reviving the Vedic sciences.

According to anatomists the left half of the brain systematically collects information, does sequential analysis and arrives at logical conclusions and results. Only the left half of the brain is activated and developed by the present Maths education being given in the schools and colleges. The right half of the human brain works with pattern recognition and has the capability of intuition. This most powerful facet of human personality — the intuitive faculty—unfortunately, remains undeveloped in most of the students. In the Vedic mathematics system, the very first step is to recognize the pattern of the problem and pick up the most efficient Vedic algorithm. Further, at each subsequent step, we recognize the pattern and complete the task by using the appropriate superfast mental working procedure, because in Vedic maths we have multiple choice available at each stage of working. This practice systematically develops the right half of the brain as well. As such by the regular practice of the most natural mental procedures of the Vedic maths system, the holistic development of the human brain automatically takes place. In a nutshell we can say that Vedic maths provides the cosmic software for the cosmic computer (the human mind), so that it is not wasted.

Vedic Mathematics originated from the Vedas, which manifest divine knowledge. Any knowledge derived from the Vedas is bound to have a touch of the divine bliss. Therefore, the very natural, easy and superfast algorithms of Vedic Mathematics bring an upsurge of joy and bliss. Owing to its very special and universal features, the Vedic maths system converts the dry and tedious maths into a playful and joyful subject, which children learn with a smile.Therefore, Vedic maths is the gift of the Veda to solve the problem of 'Maths anxiety' faced in maths education all over the world.

Important Information Regarding Vedic Maths

'It is interesting to speculate as to why the Indians found it worthwhile to pursue studies into unambiguous coding of natural language into semantic elements. It is tempting to think of them as computer scientists. Let us not forget that among the great accomplishments of the Indian thinkers were the invention of zero, and of the binary number system a thousand years before the West re-invented them. Their analysis of language (Sanskrit) casts doubt on the humanistic distinction between natural and artificial intelligence, and may throw light on how research in Artificial Intelligence may finally solve the natural language understanding and machine translation problems.'

The above citation is from Dr. R. Briggs of NASA, California, 'Knowledge Representation in Sanskrit and Artificial Intelligence,' *AI* magazine —an official publication of the American Association for Artificial Intelligence, Vol. 6, No.1, Spring 1985.

Further, the cover page of Spring 1985 issue of *AI* is decorated with the Sanskrit *ślokas* from the second chapter of *Śrīmad Bhagvad-Gītā.*

1. Vedic Mathematics is being taught at the School of Economic Sciences, London; St. James School; St. Vedast School; Mary Ward Centre, London; School of Philosophy, Australia; and several other institutions in USA, Holland, and other places.

2. Bhāratī Kṛṣṇa Tīrathjī's *Vedic Mathematics*, 1965 (now on its tenth reprinting) introduces to us the wonderful applications of the Vedic *sūtras* to arithmetic, algebra, factorizations, higher order equations, calculus, co-ordinate geometry, as also the wonderful Vedic numerical code. This book is being translated into Hindi and three foreign languages, including Latin and German. The Marathi and Gujarati versions have already been published.

3. Recently, a course was run at Mary Ward Centre, London, demonstrating that Vedic methods can be used for the entire GCE A-level Maths (equivalent to our 10 + 2).

4. On this novel Indian system of mathematics, pioneering work is being done in the U.K. The research includes applications of Vedic Mathematics to trigonometry, three-dimensional co-ordinate geometry, solution of linear differential equations, matrices, and

determinants, log and exponentials, etc. Recently, five books have been published in the U.K. giving details of the research.

5. The most interesting point to note is that Vedic Mathematics provides unique solutions in several instances where only the trial and error method is available at present. For example, Kepler's equations can be solved in just 90 seconds as compared to the present tedious trial and error method.

6. A symposium on Vedic Mathematics was held in February 1985 at Nagpur. Another symposium was held at Bhartiya Vidya Bhawan's London Centre in September 1986. The first 11 day workshop on Vedic maths and its computational potentialities was held at the University of Roorkee in August 1987. A 3 day workshop was organized in December 1987 by the Spiritual Study Group at the Banaras Hindu University, Varanasi, followed by a 4 day workshop in February 1988 at G.B. Pant University, Pantnagar. Further, a Vedic maths workshop is being organized by the Ministry of Human Resource Development and Rashtriya Veda Vidya Pratishthan at Jaipur in March 1988.*

* A Ph.D. thesis on Vedic Mathematics has been submitted by Mr. S.K. Kapoor.

Vedic maths seminars were also held during April 1988 at the Imperial College, London; Oxford University; Cambridge University; Ideal Village, Lancashire; Washington; Harvard University; Scientific Systems, Boston; the Annual NCTM meeting at Chicago; Los Angles; Stanford University, Palo Alto; University of California, Berkeley; Indiana; Goldendome MI University, Fairfield; Toronto; Mexico University; MIU, Oslo; Academy at Lily Hamer, Norway; Upsala University and Stockholm University, Sweden; Turku University, Finland; Copenhagen University and HC Oersted Institute, Denmark; Hamburg University, and Technical University, Berlin; and the Institute of Leadership, Budapest, Hungary. Further during the month of May, 1988, Vedic maths seminars were held at the Vienna University, Vienna, Austria; TM Centre, Rome; the Engineering Faculty University of Naples, and the University of Milan, Italy; MERU, Seelsberg; Geneva University, Switzerland; Paris University, France; Barcelona, Spain; MERU, Vlodrop; Amsterdam University, Holland.

Work at Roorkee

I wish to draw your attention to an aspect of the research effort at the university campus at Roorkee. Everybody who has known a little about Vedic Mathematics (VM) has derived national pride and shown a great zeal to learn more. VM kindles in our hearts the *jyotiḥ* of *śraddhā* towards India.

The Spiritual Study Group (SSG) of Roorkee University has been spreading this mathematical facet of our rich and vast scientific-cultural heritage. Since May 1985 till June 1988, SSG organized VM courses for 39 batches attended by 4300 participants, ranging from school children, teachers, university students, research scholars, field engineers, right up to university professors. All of them enjoyed learning the simple magic speed methods of solving the otherwise tedious mathematical calculations.

Since Haridwar Maha Kumbh, 1986, SSG has conducted seven VM correspondence courses in Hindi and English for home study and the eighth will start from October 1988.

Thousands of Vedic Mathematics demonstration lectures are being delivered in different parts of the world. In India, hundreds of lectures have already been given in Delhi, Punjab, Gujarat, West Bengal, U.P., M.P., Kerala, A.P., Goa, Rajasthan, Haryana and Maharashtra. Further, demonstration lectures and seminars on Vedic Mathematics have been delivered in several cities of the U.S.A., England, Canada, Mexico, Norway, Sweden, Finland, Denmark, Hungary, Germany, Italy, Switzerland, Spain, Holland in the recent past.

SSG Publications

In order to provide comprehensive details of the ancient Indian cultural heritage, SSG is publishing a series of books in Hindi and English.

Ancient VM, Pushp 1 and Pushp 2 : As a result of the research efforts by SSG, a few of the missing links have already been revived. Specifically the superfast Vedic methods of addition, substraction, tables, extensions of the base system, multiplication of three or more numbers in one line, the magic speed methods of cross-checking the correctness of any arithmetic or algebraic addition, subtraction, multiplication, division, factorization, etc., have been re-established. These two books include the new links along with the Vedic methods of multiplication, division, squaring, etc.

SSG plans to bring out further books in the ***Pushp-mala***.

VM Pushp-1 and *Pushp-2* are being translated into over thirty-six foreign languages in different parts of the world.

Video Cassettes: A series of video cassettes of lectures on different faces of Vedic Mathematics are being produced. These are available on sale or hire basis.

ILLUSTRATIONS

Vinculum

The vinculum number consists of both positive and negative digits.

The Approach: In the Vinculum process we take the complement of the digits bigger than 5 with a negative sign, and a positive carry over to the next higher order digit.

Illustration 1:

Let us take the number 9, $9 = 10-1$, therefore 9 is 1 less than 10. This we can also write as $1\bar{1}$ instead of 9. The property of the numbers is unchanged and we can add, multiply, etc. in the usual manner. However, we have to keep in mind that the $\bar{1}$ is always appearing as -1.

Illustration 2: 782893

$$782893 = 1\bar{2}\,\bar{2}3\bar{1}\,\bar{1}3$$

Let us further consider a few examples of normalizing the Vinculum number. It will be surprisingly interesting to learn that the *Nikhilam Sutra* can be used even for the normalizing process, the only difference being that the previous digit is now reduced by 1.

Illustration 3: $3\bar{3}4\bar{4}$

The complement of $\bar{3}$ is 7 and of $\bar{4}$ is 6. Further, each digit before the bar digit gets reduced by 1. So that $3 - 1 = 2$ and $4 - 1 = 3$.

Therefore $3\bar{3}4\bar{4} = 2736$

Nikhilam Methods

The second sūtra reads '*nikhilaṃ navataścaramaṃ daśataḥ*'. The literal translation of this crisp sūtra 'निखिलं नवतश्चरमं दशतः' is 'All from 9 and the last from 10.'

Now the step by step procedure is outlined below:

TYPE -1: Let us take the simple example of multiplying 13 by 11.

Step 1 : We write these numbers in the Vedic base system style and get :

13 + 3

11 + 1

We note that the base is 10 and there is only zero in the base. The positive sign indicates that both numbers are greater than the base.

Step 2 : The product will have two parts. A vertical dividing line should be drawn to demarcate these two parts.

$$\begin{array}{c} 13+3 \\ \underline{11+1} \\ | \end{array}$$

Step 3 : We vertically multiply the excess digits from base. This product gives us the righthand side portion of the answer. In this case $3 \times 1 = 3$ is the product.

$$\begin{array}{c} 13+3 \\ \underline{11+1} \\ \;\;|\,3 \end{array}$$

Step 4 : The lefthand side of the answer can be arrived at by simple adding diagonally any of the two given numbers with the deviation (deficit) of the other number from base.

Case 1 : The first number 13 is added to 1, the deficit of second number. Therefore

$$13 + 1 = 14$$

Case 2 : The second number 11 is added to 3, the deficit of the first number. Therefore

$$11 + 3 = 14$$

Note : Both these alternatives give the same result

$$\begin{array}{c} 13+3 \\ \underline{11+1} \\ 14\,|\,3 \end{array}$$

In addition to the above two alternatives the left-hand side can also be obtained by two additional procedures.

Note : A special feature of the Vedic system is the arrival of the same result in several easy ways. This feature can be utilized by the student to verify his answer step by step.

Illustration 4

$$\begin{array}{l} 111 \times 112 \\ 111 + \ \ 11 \\ \underline{112 + \ \ 12} \\ 123 \mid 132 = 123 + 1/32 = 124/32 = 12432 \end{array}$$

Note: The R.H.S. should be obtained mentally by using the Vedic method, i.e.

$$\begin{array}{l} 11 + 1 \\ \underline{12 + 2} \\ 13 \mid 2 \end{array}$$

$$\begin{array}{l} 10010 \times 10101 \\ 10010 + 10 \\ \underline{10101 + 101} \\ 10111 \mid 1010 \end{array} \qquad \begin{array}{l} 1099 \times 1001 \\ 1099 + 99 \\ \underline{1001 + 01} \\ 1100 \mid 099 \end{array} \qquad \begin{array}{l} 103 \times 13 \\ \text{Taking base as 10} \\ 103 + 93 \\ \underline{\ 13 + 3} \\ 106 \mid 27^9 = 106 + 27/9 \\ \qquad\quad = 133/9 \\ \qquad\quad = 1339 \end{array}$$

$$\begin{array}{l} 985 \times 995 \\ 985 - 15 \\ \underline{995 - 5} \\ 980 \mid 075 \end{array}$$

Ūrdhva Method

Illustration 5: Let us consider the multiplication of two two-digit numbers.

$$\text{Multiply } 13 \times 21 \qquad \begin{array}{r} 21 \\ \times\ \underline{13} \\ \underline{\ \ 3} \end{array}$$

Step 1 : We simplify the first column

$$\begin{aligned} R_1 &= U_1 \times L_1 \\ &= 1 \times 3 = 3 \end{aligned}$$

$R_1 = 3$ is the extreme right part of the answer and is written below the first column.

Step 2 : We operate the first column with each of the subsequent columns. We include the second column which, on simplification, gives the next digit.

$$R_2 = U_1 \times L_2 + U_2 \times L_1$$
$$= 1 \times 1 + 2 \times 3 = 7$$

$R_2 = 7$ is the result at the next level and is written at the tenth level.

With this we complete the operation of the first column.

Step 3 : We eliminate the first column and simplify the remaining columns (here only one column is left): therefore

$$R_3 = U_2 \times L_2$$
$$= 2 \times 1 = 2$$

$R_3 = 2$ is the resulting digit at the next level (hundred in this case). The result is written into appropriate level.

$$\begin{array}{r} 21 \\ \times \underline{13} \\ \underline{2/7/3 = 273} \end{array}$$

Cubing

Introduction: We can do the cubing of the numbers using the sub-*sūtra* *Ānurūpyeṇa,* which means 'proportionately'. The following sections illustrate the step by step procedure of computing the cube of numbers.

In the general case the first term is cube of the first part (say a^3) and each of the subsequent three terms is obtained by multiplying the first term by the second part (b) and dividing by the first part. That is, the subsequent terms are the G.P. of the ratio of 2 digits. Then we double the 2nd and 3rd terms, add and carry.

Illustration 6: 23^3.

$$\begin{array}{lllll} 23^3 = & 8 & 12 & 18 & 27 \\ & & 24 & 36 & \\ \hline & 8 & 36 & 54 & 27 \\ = & & 8\ 36 & 54 & 27 \\ = & & 12 & 1 & 67 \end{array}$$

1st term $= 2^3 = 8$
2nd term $= 8 \times 3/2 = 12$
etc. 3rd term $= 12 \times 3/2 = 18$
4th term $= 18 \times 3/2 = 27$

Illustration 7: 34^3

$$\begin{array}{lllll} 34^3 = & 27 & 36 & 48 & 64 \\ & & 72 & 96 & \\ \hline & 27 & 108 & 144 & 64 \end{array}$$

$= \quad 27 \quad 10^8 \quad 10^4 \quad 6^4 = 3\,9\,3\,0\,4$

The cube of a three-digit number can be obtained in two steps.

Simple Equations

Using the simple Vedic sūtras it is possible to solve a large class of equations directly through mental computations. Several types have been covered by the Swami Ji in his book (Ref.4). Just a couple of examples are presented here.

Illustration 8: $\dfrac{5x+3}{2x+7} = \dfrac{4x+9}{7x+5}$

Since $(5x + 3) + (4x + 9) = (2x + 7) + (7x + 5) = 9x + 12$,

We can at once say, using *Shunyam Sūtra,*

$\therefore \; \underline{9x + 12 = 0}$

Whence $x = -12/9 = -4/3$

But we are dealing with a quadratic; its second solution can be obtained as follows. Write the equation as

$$\frac{N_1}{D_1} = \frac{N_2}{D_2}$$

Now $N_1 \sim D_1 = (5x + 3) - (2x + 7) = 3x - 4$

and $N_2 \sim D_2 = (7x + 5) - (4x + 9) = 3x - 4$

(Where ~ means that we subtract the smaller term from the larger and for this purpose we can give the coefficient of x a greater weight than the constant term.)

$\therefore$ the second solution is

$\underline{3x - 4 = 0}$

(2) $\dfrac{5x-1}{2x+7} = \dfrac{8x+9}{11x+1}$

$\therefore \quad 13x + 8 = 0$

i.e. $x = -8/13$

and $3x - 8 = 0$;

Evaluation of Determinants (Ref. 1,8 & 10)

Illustration 9: Evaluate the following determinant:

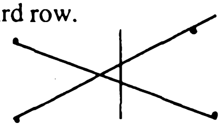
$$E = \left|\begin{array}{ccc} -26 & -11 & 17 \\ \hline -3 & 7 & 1 \\ 2 & 4 & 3 \\ \hline 5 & 8 & -4 \end{array}\right|$$

The co-factors of the first two rows are computed by using *Ūrdhva Sūtra* and written at the top.

$-3 \times 4 - 2 \times 7 = -26$ etc.

Further, the vertical and crosswise sūtra is used and the co-factors (written in the top row) are multiplied by the third row.

$$E = (-26) \times (-4) + (-11) \times 8 + 17 \times 5$$
$$= 277$$



The Ūrdhva method can be extended to evaluate the determinants of size 4×4 and 6×6 also. The higher size determinants can be computed by first reducing their size, using the pivoting method.

Simultaneous Linear Equations (Ref. 1, 4, 8 & 10)

For 3 equations in 3 unknowns we can use the partitioning technique, and compute the 6 co-factors of two portions (written outside) using *Ūrdhva Sūtra* (see the previous section). Further, the four 3×3 determinants are computed mentally by using Ūrdhva, extending them by using the appropriate column. In the third step the values of the unknown variables are obtained directly. The whole working is shown below :

Illustration 10:

	xD	yD	zD		D	
−8	4x	−2y	+ 3z	=	8	−5
14	2x	−3y	+z	=	1	−17
13	3x	+2y	−z	=	3	4

$zD = 8 \times 13 - 14 \times 1 - 8 \times 3 = 66$ etc.

$\therefore$ D = 33 ; xD = 33; − yD = −33; zD = 66

Whence x = y = 1 ; z = 2

This method can be further extended to solve for four unknowns. Further, using the pivoting technique it is now possible to solve with equal

ease and simplicity even eight or ten simultaneous equations. The total working space required is hardly two pages.

Transcendental Equations

The method described in the computations of trigonometric functions can be applied to the solution of transcendental equations.

Illustration 11: $x + \sin x = 1$

$$x + x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots = 1$$

$$x + 0.5 + \frac{x^3}{12} - \frac{x^5}{240} + \frac{x^7}{2.7!} \ldots\ldots$$

0.5		0.5				
$x^3/12$	12		1_3	1_1	1_2	2_2
$-x^5/240$	2				$\bar{1}_1$	$\bar{5}_{\bar{1}}$
x	=	0.5	1	1	0	$\bar{3}$
$(x^2$	=	$.3_{\bar{5}}$	$\bar{4}_0$	1	1_2	$1_1)$

The method is quick and efficient if x is small, and can be extended to any number of decimal places.

Kepler's Equation

This is an equation of great importance in positional astronomy. The equation is $M = E - e \sin E$

Where M, E are the mean and eccentric anomalies respectively of a planet in its orbit, and e is the eccentricity of the orbit.

We shall take M = 0.30303 radians and e = 0.016722—which is the eccentricity of the Earth's orbit. We want to find E.

Illustration 12: $0.30303 = E - 0.16722 \sin E$

$$E = 0.30303 + 0.016722 \left(E - \frac{E^3}{3!} + \frac{E^5}{5!} - \ldots\ldots\right)$$

M	0 3 0 3 0 3 0 0	1
eE	$1_{\bar{4}}^{\bar{5}}\ {}_{3}^{2}\ {}_{\bar{6}}^{\bar{5}}\ {}_{1}^{2}\ {}_{1}^{1}\ \bar{3}$	2
– eE3/3! 6	$\bar{1}_{\bar{1}}2_{\bar{1}}^{\bar{1}}\ {}_{3}^{\bar{5}}\ {}_{0}$	3
eE5/5! 2	4_0	4
E =	$0\,.\,3\ 1\ \bar{2}\ 1\ 0\ 1\ 0$	5

$$\left(E^2 = .\,1_{1}0_{\bar{4}}\ {}^{\bar{5}}_{\bar{1}}\ {}^{\bar{1}}_{2}\ {}^{3}_{\bar{4}}\right)$$

Where e = 0.16722 = $0.02\bar{3}\bar{3}22$

Bring 0.3 into the answer.

Row 2 : We multiply e and E : $CP\left[\begin{smallmatrix}2\\3\end{smallmatrix}\right] = 6 = 1\bar{4}$

Bring 1 into the answer.

Row 2 : $CP\left[\begin{smallmatrix}2\bar{3}\\31\end{smallmatrix}\right] = \bar{7}, \bar{7} + \bar{4}\ \bar{0} = \bar{5}_3$

Bring down $\bar{2}$ into the answer etc. For further details refer (Ref. 1, 8 & 10).

At present, in the current mathematics, we have only the trial & error method available.

Non Linear Differential Equations

The method of series solution of differential equations is well known. This when combined with the Vedic mathematical system consisting of sixteen sūtras as re-structured by Shri Bharati Krishen Tirthji makes this an immensely powerful approach. This procedure can be used to handle a wide range of problems, particularly those where the boundary conditions are given at the origin or at a single point.

Illustration 13 : Solve $2y + (y')^2 = 13 + 18x + 6x^2$,

Where $y(1) = 6$

The boundary conditions not being given at the origin, we have a choice between expanding y as a power series in (x – 1), or changing to a new independent variable z = x – 1. Both approaches amount to the same thing.

Following the former course, we have :

$2y + (y')^2 = 37 + 30\,(x - 1) + 6\,(x - 1)^2$,

Where $y(1) = 6$

The solution now proceeds :

$$\begin{array}{l} \quad 6 \quad 5 \qquad\qquad 1 \qquad\qquad 0 \qquad\qquad 0 \\ y = A + B(x-1) + C(x-1)^2 + D(x-1)^3 + E(x-1)^4 + F(x-1)^5 + ... \end{array}$$

$$\begin{array}{l} y' = B + 2C(x-1) + 3D(x-1)^2 + 4E(x-1)^3 + 5F(x-1)^4 + 6G(x-1)^6 + ... \\ \quad 5 \quad 2 \qquad\qquad 0 \qquad\qquad 0 \end{array}$$

We first find $A = 6$, then $B^2 = 25$

This leads to two solutions, one with $B = +5$ (given above), and one with $B = -5$.

$\therefore$ i.e., the two solutions are :

$\underline{Y = 6 + 5\,(x-1) + (x-1)^2}$ — (exact solution)

& $\underline{Y = 6 - 5\,(x-1) - 2\,(x-1)^2 - 0.2\,(x-1)^3 - 0.11\,(x-1)^4 ...}$

Illustration 14 : Solve $y + 2y'\,y'' = \exp(x)\cos 2x$ .....
Where $y(0) = 1$, and $y'(0) = 1, 1, y'''(0) = 0$

A power series expansion for exp(x) cos 2x can be obtained from MacLaurin's Theorem. Alternatively, as shown below, write down the expansion for exp(x), and below it write the expansion for cos 2x, keeping like powers in the same column:

$$y + 2y'\,y'' = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + ...\right) \text{ times}$$

$$\left(1 + 0 - \frac{4x^2}{2!} + 0 + \frac{16x^2}{4!} + 0 + - \frac{64x^6}{6!} + ...\right)$$

The right-hand side coefficients of the successive powers of x can now be worked out as needed, using the 'vertically and cross-wise' procedure, i.e., the solution is,

$$\underline{y = 1 + x - \frac{1}{8}x^4 - \frac{11}{240}x^5 - \frac{7}{840}x^6 .....}$$

Integro Differential Equations

Illustration 15: Solve $y^2 + \frac{2(dy)^2}{dx} + y \int_0^x ydx = 4 + 4x + 60x^2$

$8x^3 + 9x^4 + 3x^5$

Where $y(0) = 2$, $y'(0) = 0$

Assume

$y = a^2 + bx0 + cx^2 + dx^3 + ex^4 + fx^5 + gx^6 + \ldots$

$\therefore\ y' = b0 + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5 + 7hx^7 + \ldots$

$$\& \int_0^x ydx = 0 + \frac{2x}{2} + \frac{b}{2}x^2 + \frac{c}{3}x^3 + \frac{d}{4}x^4 + \frac{e}{5}x^5 + \frac{f}{6}x^6 + \ldots$$

Boundary conditions lead to $a = 2$ and $b = 0$, as shown above. Succeeding steps involve the solution of simple equations, and

$y = 2 - 3x^2$ is one solution.

Vedic Numerical Code

It is matter of interest to note that Vedic scholars of ancient times did not use figures (for big numbers) in their numerical notations but preferred to use the Sanskrit alphabet to represent various numbers. This was perhaps done to facilitate the mathematical knowledge stored and transferred on to subsequent generations by way of a language understood by mankind. Thus the preservation of their knowledge was better managed and understood by subsequent followers. Sūtras and Sanskrit verse are in fact treasures of such kind which can be easily decoded and assimilated even to the modern world.

In this context, it is interesting to point out the honour being given to the Sanskrit language by the American Association for Artificial Intelligence. The full cover page of the *AI* magazine (volume 6 No. 1 spring 85), an official publication of the American Association for the artificial intelligence is decorated with Sanskrit shlokas. This particular issue also contains a research article of ten pages by Dr. Rick Briggs, who is a computer scientist at NASA, American Space Research Centre, California, on 'Knowledge Representation in Sanskrit and Artificial Intelligence'.

The Code: The code system in Hindi could be written down as :

क	ट	प	1	य
ख	ठ	फ	2	र
ग	ड	ब	3	ल
घ	ढ	भ	4	ब
ङ	ण	म	5	श
च	त		6	ष
छ	थ		7	स
ज	द		8	ह
झ	ध		9	
		क्ष	0	

Note : Further न and ञ are also coded as 5

Illustration 16 :

कंसे क्षामदाहखलैर्मलैः

Here Kaṃse means 17, *Kṣāma-dāha-khalairmalaiḥ* means 0 5 8 8 2 3 5 3.

Thus $\frac{1}{17}$ is obtained by 0 5 8 8 2 3 5 3 x 9 9 9 9 9 9 9 9

i. e. $\frac{1}{17}$ = . 0 5 8 8 2 3 5 2 9 4 1 1 7 6 4 7

Lastly a *Śloka* for obtaining the value of $\pi/10$.

गोपीभाग्यं मधुव्रातशृङ्ग्योदधिसन्धिगः।
खलजीवितखातावगलहालारसंधरः॥

This *Śloka* has two sets of literal meaning—the first in praise of Lord Krishna and the second for Lord Shiva. The third meaning (after decoding) gives the value of $\pi/10$ to 32 places of decimals as below:

$$\frac{\pi}{10} = .31415926535897932384626433832792\ldots\ldots$$

Further research is revealing very interesting results. For instance, the Sacred *Gāyatrī Mantra*, when decoded, yields the sacred number 108.

Observations : Dr. V.P. Dalal (of Heidelburg University, Germany) after learning about the merits of the above *Śloka* commented as follows: "It shows how deeply the ancient Indian Mathematicians penetrated, in the subtlety of their calculations, even the Greeks had no numerals above 1000 and their multiplications were so very complex, which they performed

with the help of the counting frame by adding so many times the multiplier; 7×5 could be done by adding 7 on the counting frame 5 times," etc.

Plane and Spherical Trigonometry

The Pythagorean triples which are based on the theorem of Pythagoras, when combined with the Vedic system of mathematics are of great practical use. They link the two main branches of mathematics, number and geometry. They have useful applications in trigonometry, co-ordinate geometry, etc., and form a unique thread through many areas of mathematics. The British mathematician Kenneth Williams in his recent book *Triples* has shown very interesting applications of Vedic maths and Triples in solving problems with great speed, problems of trigonometry transformations in a plane, 2D & 3D co-ordinate geometry, 3D transformations, solution of plane and spherical triangles, and prediction of planetary positions.

Illustration 17 : 3, 4, 5; 6, 8, 10; 5, 12, 13 all have the property that the sum of the squares of the first two numbers is equal to the square of the third, i.e. $3^2 + 4^2 = 5^2$ etc.

Any three numbers having this property are called a *triple.*

In general, the parabola $y^2 = 4ax$, in which the time and the point are 2a apart.can be constructed by triples in which 1st and 3rd elements differ by 2a.

Further, an experimental set-up can be prepared after going through the planet-finder and its construction as described by Kenneth. With the help of this set-up we can very easily find the position of any planet at any time of any year after performing the simple experiment.

Discussion

The essence of the Vedic system is that it is simple, direct, one-line and mental. The Vedic sutras are applicable to a wide variety of problems. They are very easy to understand and a delight to use.

1. The property of numbers is very extensively exploited in Vedic mathematics, particularly in the field of computations. The Vedic approach has many special methods—indeed many more than conventional methods. This large flexibility of methods finds itself reflected in the mind, when approaching a problem from the Vedic

viewpoint. This indeed is the great benefit of the approach. One wonders at the supreme simplicity and ease of the Vedic method, which is lacking in most of our usual methods.

2. Vedic maths is perfectly adapted to oral teaching and mental calculation. The Sūtras, of course, demand regular practice of simple problems. They are perfectly in tune with the oral Vedic tradition supplemented by terse statements of important points called Sūtras, which act as an aid to the memory.

3. Even a little exposure to the Vedic maths approach, coupled with some practice, clearly shows us that we are dealing with an entirely new and direct way of thinking.

4. In May 1985 when the first Vedic Mathematics course was being organized at the University campus, one of our students from Nepal, Bodhswarup (of B.E. Final Year Civil)who had just heard about the Vedic Mathematics book by Śaṅkarācārya Jī Mahāraja knew about the Vedic general method of multiplication (based on the *Ūrdhva Sūtra*). He had a perfect mastery over this method. He also introduced to us the *Śuddha upasūtra* and its use for the Vedic method of addition, subtraction, and compound addition and subtraction. Mr. Bodh had learnt these Vedic methods from his spiritual Guru who also was not aware of Śaṅkarācārya Jī's publication.

Further, Dr. Khanna of B.H.U. also informed us about his working knowledge of the general Vedic method of multiplication.

Dr. Ram Chandra from Jodhpur, who was not aware of Śaṅkarācārya Jī's publication, explained to us the Vedic method of checking the results of any multiplication. Later we found the procedure to be straight application of the *Guṇita Sūtra.*

In Bhāskarācārya's *Līlāvatī*, one of the methods of multiplication uses the deviation of the number from the base, which is very close to the Nikhilam method of Vedic maths.

The alpha numeric coding system is based on the verse called *Kaṭādi* (कटादि)*Sūtra* in ancient *Phalita Jyotiṣa* (फलित-ज्योतिष)—predictive astrology. There are several applications of the same. For example, one of the matrices of (size 8×12) 96 elements giving the relative weightage, due to

the inter-planetary positions, has been codified in a simple four-line Sanskrit verse. All the 96 elements of this matrix can be obtained at any time by simply decoding the verse.

Further, Dr. Rahul from Gorakhpur has recently decoded the *Gāyatrī* using the same key. The first part of the *Śatākṣara Gāyatrī Mantra* has 24 letters. The numerical value of this adds up to the auspicious number 108. Many persons are using the Vedic addition process with slight variations.

All these indicate clearly that the different facets of Vedic Mathematics have been scattered due to mass destruction and killings at the ancient Vedic seats of learning. There is need to do further extensive research both in applications and cross-linkage. But whatever is available today is sufficient to make an excellent start both at the teaching as well as at the research level.

5. Vedic Mathematics, owing to its multi-choice facility, provides wonderful opportunities for the development of the innovative and research faculty of young students. A large number of Vedic maths participants have been coming forward with very interesting and useful links. The age and qualifications are no barrier in this process. The extensions of the Nikhilam method for multiplying three or more numbers and the use of different bases and subbases have all been provided by one of our undergraduate students.

   It will be interesting to point out the recent research work of Mr. Sanjay, who has just obtained his Engineering Diploma. Mr. Sanjay has very successfully extended the Vedic method of cubing, to find any power of a given number even beyond computer capability. Further, he has found that the Ūrdhva method of solving linear simultaneous equations can be used for solving even 8 or 10 unknowns using the Pivoting technique. The whole working can be completed in just two pages.

6. Vedic maths primarily for mental calculation and has the potential of developing the human computer to a wonderful level where the results of the problems flow very naturally, with the least amount of effort, in the typical Vedic way. This approach provides a corrective methodology to the problem of mental slavery to calculators.

Further, Vedic algorithms have considerable potential for automatic computations and small computers can handle much bigger problems owing to the ease and simplicity of the processes and lesser memory requirements:

Let us learn, teach and preach Vedic mathematics which children can learn with smiles on their face and joy in their hearts.

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On Factorization and Partial Fractions

WAZIR HASAN ABDI

At the outset I must confess that I am not a student of Sanskrit but as I have been involved in teaching mathematics for over four decades at all levels from class VIII onwards, no wonder any principle, method, technique, recipe, aid-to-memory, history or philosophy related to mathematics would lie within the sphere of my interest, even if marginally. Naturally I found Swamiji's monograph very thought-provoking especially the chapters dealing with factorization and allied topics.

The monograph devotes chapter VII to the factorization of what is called simple quadratics $ax^2 + bx + c$. After examining the current method, the procedure laid down reads: 'Split the middle coefficient into such parts that the *ratio* of the first coefficient to the first part is the same as the ratio of the second part to the last coefficient.' (p.87)

As an illustration the quadratic $2x^2 + 5x + 2$ is considered: Here 5 is split into 4 and 1 so $2:4 = 1:2$. Thus $x + 2$ is one factor and the second factor is obtained by dividing the first coefficient of the quadratic by the *first co-efficient* of the factor and the *last coefficient* of the quadratic by the last coefficient of that factor. In other words the second binomial factor is obtained thus : $\frac{2x^2}{x} + \frac{2}{2} = 2x + 1.$

Thus we say : $2x^2 + 5x + 2 = (x + 2)(2x + 1)$. (pp 87-88)

Ignoring the loose use of the term 'coefficient' the method outlined amounts to finding u and v such that : $u + v = b$, $a:u = v:c$. It is not clear how this procedure will have greater appeal to the school children who are familiar with the technique: $u + v = b$, $uv = ac$. In fact the current method involves splitting ac into factors which is less cumbersome than establishing ratios after splitting b into two parts.

It may further be noted that Swamiji's technique completely ignores quadratics which involve 'perfecting the square' or the type $x^2 - b$ where b is not a perfect square but positive, or $x^2 + b$ where b is positive, perfect square or not. Nowhere does the chapter on the so-called simple quadratics give any indication that the numbers involved could be rational, irrational or complex conjugate even when a, b, c are integers. This impression is confirmed by the nineteen examples given in chapter VII where only

integers appear as the constant term or the coefficient of x or its higher powers. This trend is further strengthened when some quadratics are classified as *factorless* (which obviously can be split into factors with constant terms or coefficients of x, being complex or irrational). (p.102)

Harder Quadratics

The chapter devoted to factorization of homogeneous polynomials of second degree in three variables describes the *lopana-sthāpana* method. The technique is simple and can be expressed as follows :

Let $P_2(x, y, z)$ be a homogeneous polynomial of second degree. Then

$$P_2(x, y, o) = (a_1x + b_1y)(a_2x + b_2y)$$
$$P_2(x, o, z) = (a_1x + c_1z)(a_2x + c_2z)$$
$$\therefore \quad P_2(x, y, z) = (a_1x + b_1y + c_1z)(a_2x + b_2y + c_2z)$$

No explanation is given for the validity of the method but it works in many cases. For 4 dimensions consider $P_2(x, y, z, w)$. Then

$$P_2(x, y, o, o) = (a_1x + b_1y)(a_2x + b_2y)$$
$$P_2(x, o, z, o) = (a_1x + c_1z)(a_2x + c_2z)$$
$$P_2(o, y, o, w) = (b_1y + d_1w)(b_2y + d_2w)$$

Hence $P_2(x, y, z, w) = (a_1x + b_1y + c_1z + d_1w)(a_2x + b_2y + c_2z + d_2w)$.

Our assumption here is that the coefficients in the factors could be complex, while the examples in Swamiji's monograph involve only integral coefficients and constants. The method has been shown to work even with linear and constant terms also. However snags appear in such cases where the rule is applied mechanically. For example it has been shown that when

$$P_3(x, y, z) = 3x^2 + 7xy + 2y^2 + 11xz + 7yz + 6z^2 + 14x + 8y + 14z + 8,$$

then $P_3(x, o, o) = 3x^2 + 14x + 8 = (x + 4)(3x + 2)$

$$P_3(o, y, o) = 2y^2 + 8y + 8 = (2y + 4)(y + 4)$$
$$P_3(o, o, z) = 6z^2 + 14z + 8 = (3z + 4)(2z + 2)$$

Hence $P_3(x, y, z) = (x + 2y + 3z + 4)(3x + y + 2z + 2)$ (p.92)

It is not clear why 7xy, 11xz and 7yz have been *completely ignored* in this computation and why instead of eliminating (equating to zero) only one variable two variables have been eliminated at a time. Possibly the method would have *completely failed* because, for example,

$$\begin{aligned} P_3(x, y, o) &= 3x^2 + 7xy + 2y^2 + 14x + 8y + 8 \\ &= (3x + y)(x + 2y) + 2(7x + 4y + 4) \end{aligned}$$

which can not be expressed in two linear factors. Maybe the factors $x + 2y + 3z + 4$ and $3x + y + 2z + 2$ have been multiplied together first to obtain the concerned polynomial.

Partial Fractions

Partial fractions involve decomposing the denominator into irreducible factors. Chapter XXII is concerned mainly with the case when the factors are all linear and unrepeated. It has been stated that 'the current systems have a very cumbrous procedure but which *Parāvartya Sūtra* tackles very quickly with its well-known MENTAL ONELINE answer process'. (p. 186)

As an illustration, $E = \dfrac{3x^2 + 12x + 11}{(x+1)(x+2)(x+3)}$ has been considered equal to

$\dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{x+3}$.

The current method leads to three simultaneous equations in three unknowns A, B and C, whose solution yields the required result, 'In the Vedic System', however, for getting the value of A,

(i) we *equate its denominator to zero* and thus get the paravartya value of A (i.e. −1)
(ii) and we mentally substitute the value −1 in the E (but without the factor which is A + s denominator on the right hand side) and
(iii) we put this result down as the value of A. Similarly for B and C.

'All this work can be done *mentally*; and all the laborious work of deriving and solving three simultaneous equations is totally avoided by this method.'

There is no doubt that the procedure given in this chapter is very time-saving and is in fact useful even in the case of a large number of linear

factors. But it does not seem justifiable to call it a Vedic method as opposed to the current method. In the popular method there is a logical explanation: both the sides are multiplied by (say) x + 1 and thus

$$\frac{(x+1)(3x^2+12x+11)}{(x+1)(x+2)(x+3)} = (x+1)\left(\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}\right)$$

or $$\frac{3x^2+12x+11}{(x+2)(x+3)} = A + (x+1)\left(\frac{B}{x+2} \quad \frac{C}{x+3}\right)$$

Partial Fractions and Integration

In problems of integration of rational functions it has been suggested that the integrand should be split by *parāvartya*, first factorizing the denominator. Three examples have been given to 'make the procedure clear' Example (2) reads:

Integrate $\dfrac{x^2-7x+1}{x^3-6x^2+11x-6}$

$$\because \text{(By } \textit{parāvartya}) = \frac{x^2-7x+1}{(x-1)(x-2)(x-3)}$$

$$= \frac{-5}{2(x-1)} + \frac{9}{x-2} - \frac{11}{2(x-3)}$$

$$\therefore \int \frac{(x^2-7x+1)\,dx}{x^3-6x^2+11x-6} = \int \left(\frac{-5}{2(x-1)} + \frac{9}{x-2} - \frac{11}{2(x-3)}\right)^{dx}$$

$$= -\frac{5}{2}\int \frac{dx}{x-1} + 9\int \frac{dx}{x-2} - \frac{11}{2}\int \frac{dx}{x-3}$$

$$= -\frac{5}{2}\log(x-1) + 9\log(x-2) - \frac{1}{2}\log(x-3) \text{ (p. 192)}$$

This procedure will not appear to a student any different from the current method, and such simple cases where the denominator can be split into unrepeated linear factors with integral or even rational or irrational coefficients no difficulty will arise. But one question still remains unanswered: Why is $\int \frac{dx}{x-a} = \log(x-a)$ and how has the logarithmic function been defined in Vedic (or even our classical mathematical) literature? (p. 102)

Conclusion

In conclusion, I must say that Swamiji's monograph has done a singular service: it has reminded us that we have neglected our rich mathematical heritage for so long and it is our duty to know our past, learn from our failings, throw out the dead wood and take up from where our ancestors left. To my knowledge, the first serious attempt in this direction has been made by Bibhuti Bhushan Datta and Avadhesh Narain Singh in the two volumes of *Hiṇdu Mathematics*, supplemented by Kripa Shankar Shukla in the third volume, which still remains unsurpassed. The great task pioneered at Lucknow continues at the school created by the Department of Mathematics and Astronomy, University of Lucknow.

Operational Techniques

C. SANTHAMMA

Based on the operational techniques propounded by Śrī Jagadguru Śaṅkarācārya of Puri, quite a large number of mathematical problems could be solved in an elegant way which appear to be novel in their simplicity, less laborious, less time-taking, and as such one can think of such methods to be introduced in the curriculum at different levels.

Vedic mathematics should be studied by the Computer Scientists to devise a Vedic Computer.

It should also be subject to further research, exploring the possibilities of its sūtras to elicit more information, and to make mathematical operations less tedious than they are. Each of these sutras, when applied to proper operations, are yielding exciting results. As an example of such application, general multiplication and division are explained.

General Multiplication: The sūtra reads as *Ūrdhva tiryagbhyām.*

As applied to two digit numbers, say,

$$\begin{array}{r} 23 \\ \underline{46} \end{array} \uparrow$$

The first step is *Ūrdhva*, i.e. 6x3 = 18 and has to be shown as

$$\begin{array}{r} 23 \\ \underline{46} \\ 18 \end{array}$$

The second step starts with the first column. Perform the operation as shown:

$$\begin{array}{llc} & 6x2 = 12 & 23 \\ & & \times \\ & \underline{4x3 = 12} & 46 \\ \text{add up} & \quad 24 & \end{array}$$

The second step results in after adding 1 to 24

$$\uparrow \begin{array}{r} 23 \\ \underline{46} \\ 58 \\ 21 \end{array}$$

The third step is the *Ūrdhva* Multiplication of 2 and 4

$$2 \times 4 = 8$$

and then is added to 2

$$\begin{array}{r} 23 \\ \underline{46} \\ 1058 \\ \underline{21} \end{array}$$

This is one line method.

It is surprising to note that the multiplication can be done from left side as well, and is as follows:

The first step :

$$\begin{array}{r} 23 \\ \underline{46} \\ 8 \end{array}$$

The second step :

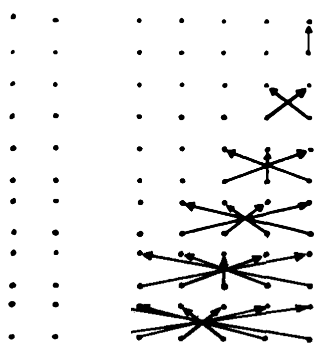
$$\begin{array}{rl} 4 \times 3 = 12 & 23 \\ \underline{6 \times 2 = 12} & \underline{46} \\ 24 & 84 \\ & 2 \end{array}$$

The third step:

$$\begin{array}{r} 23 \\ \underline{46} \\ 848 \\ \underline{21} \\ 1058 \end{array}$$

The method of multiplication from left to right is really novel. A comparison of the existing method with the right to left method evidently brings out the differences in the working methods.

This can be generalized as follows considering an 8-digit by 8-digit multiplication,



1st step

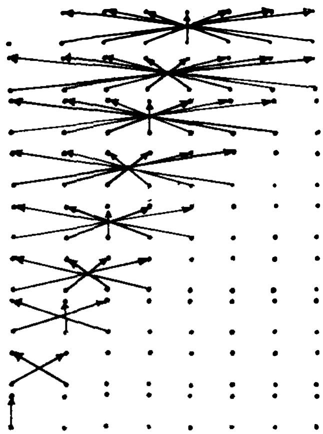
2nd step

3rd step

4th step

5th step

6th step



7th step
8th step
9th step
10th step
11th step
12th step
13th step
14th step
15th step

Verification with the existing method:

23 can be written as $2 \times k + 3$
$k = 10$

46 46 can be written as $4 \times k + 6$

$2x + 3$

$4x + 6$

$8x^2 + 24x + 18$

$\begin{array}{cc} 2 & 3 \\ 4 & 6 \end{array}$

In general

$ax^2 + bx + c$ $\quad \begin{array}{ccc} a & b & c \\ d & e & f \end{array} \quad \begin{array}{ccc} a & b & c \\ d & e & f \end{array} \quad \begin{array}{ccc} a & b & c \\ d & e & f \end{array}$

$dx^2 + ex + f$

$adx^4 + (ae + bd)\,x^3 + (af + dc + be)\,x^2 + of$ $\quad \begin{array}{ccc} a & b & c \\ d & e & f \end{array}$

is a highly self explanatory verification.

General Division

Dhvajanha - Sūtram

	3	5588	5
8			

3 is considered to be the flag digit

The number of digits to be kept aside is equal to number of digits in the flag.

Operation

	3	55 88	5
8		7 + 4	3
		6 7 3	26 Remainder

1. Treat 55 as the first dividend and divide by 8.
2. 7 is the remainder. Consider 78 as the next 'dividend, and subtract from it, the first quotient multiplied by the flag and subtract from the dividend. The result is divided by 8.
3. The process is continued till all the numbers of the original dividend are exhausted.
4. Q 673, R 26 is the answer.

Existing Method

83) 55885 (673
498
608
581
275 673 = Q
249 R = 26
26

The differences between the two methods are very clear.

The First Method

1. Consider the number of dividend digits to be equal to the number of divisor digits for operation throughout, and hence divide first only with the non-flag digit.
2. New dividends are formed, out of quotients and remainders and part of the original dividend and the flag under different operations.

These methods are very elegant and should be given due publicity for appreciation and implementation.

Mathematics for Joy

ISHWAR BHAI PATEL

If one wants to really enjoy mathematics, one must study *Vedic Mathematics* by Jagadguru Śaṅkarācārya Bhāratī Kṛṣṇa Tīrthaji. The short-cut *sūtras* offer the prospect of getting mathematical solutions quickly and make it possible for young people to demonstrate to their parents multiplication of a 20-digit number by a 19-digit number, without using pen and paper. To make Vedic Mathematics popular, we decided therefore to publish *Vedic Ganit Vidya* in Gujarati, launched a series of training programmes for initiating secondary level students into Vedic Mathematics. With the help of a newspaper columnist, we projected in a daily what the unparalleled and novel features of Vedic Mathematics are: multiplication, division, decimals, fractions, factors and the like. This gave a great impetus to the learning of Vedic Mathematics.

The basic understanding of the application of two *sūtras* and the quickness with which results could be obtained, besides restoring their confidence, encouraged the students to take on more problems.

The same exercise repeated on secondary teachers convinced us of the efficiency of the Vedic mathematical sūtras in making mathematics a matter of joy and ease.

We solicited then the reaction of the various groups to find out what impressed them most about the *sūtras* and how Vedic Mathematics could be incorporated in the mainstream of mathematics curriculum. The essence of what they said was that these *sūtras* were short, easy to remember, and easy to apply, with no cumbersome process of several steps before arriving at the result. In the sutric method the pleasure and delight of noting down the answer and its quick verification of thirty, forty digits straightaway had a satisfaction of its own. These *sūtras* inspired a sort of confidence in one's ability to tackle mathematical problems, which is necessary for a sustained interest in mathematics. The speed, distinctness, and directness with which one arrived at the answer made Vedic Mathematics a class by itself. Also the wide applicability of Vedic Mathematics assigns it a distinct place in the ever-expanding field of knowledge. Thus it is clear that a research project should be initiated or a research department established to explore the unknown frontiers of Vedic mathematics. This department may

undertake a reconciliation between the formal mathematics curricula and the Vedic formulae, absorbing the best of both, and evolve a new concept of mathematics which will preserve the best in our ancient heritage and at the same time make the most modern available in the same classroom package.

Some expressed the opinion that the lovers of Vedic Mathematics should direct their energies also to evolve this new blend of mathematical studies that will prove a great boon to generations ahead.

The ancient Vedic sages have, in no uncertain terms, declared mathematics to be the cornerstone of all scientific knowledge and the whole world, even today, acknowledges that they are grateful to India for the discovery of the zero and numbers. How the sages came upon the zero and numbers is a mystery. Perhaps it was intuition, which is the result of patient observation, hard deduction, and intense thinking. One of the great scientists, Einstein, acknowledged the place of intuition in all scientific and other epoch-making discoveries in his book *Science and Intuition.* Einstein admitted that intuition plays its part in momentous discoveries. In our living memory Mahatma Gandhi acknowledged what part intuition played in his historical decision to call the British to quit India in April, 1942.

For more than seven years Jagadguru concentrated on Vedic Mathematics and presented 16 *sūtras* and 13 sub-*sūtras* which cover a whole range of mathematical operations from arithmetic to analytical cones. That is the work of a genius working on intuition. Even Ramanujam, the mathematical genius, born only a hundred years ago, during the life-span of only 32 years, has left behind 4000 mathematical formulae which the mathematicians the world over are trying to decipher. His solution of pie (π) puzzled many, till recently on a computer, they worked out its answer to 17 million digits.

We owe to Jagadguru that we lay bare all the ramifications of Vedic Mathematics for the present and further generations with the use of sophisticated equipment like computers. Let us hope we prove equal to the task and to the challenge.

How easy it is to learn mathematics the Vedic Mathematics way, would be evident from the fact that Shri Manish Soni, a student of Standard XII, took to Vedic Mathematics. He found it so absorbing that he could not resist the temptation of sharing this with his friends. It would be interesting to note that he introduced 2000 high school students in Baroda and 1000 in Ahmedabad to Vedic Mathematics. The peer group psychology helped

us immensely in eliciting a warm response from the students. When he started presenting Vedic Mathematics to secondary teachers of mathematics at Idar, people wondered how he would keep the teachers on their seats. For three hours, he went on presenting Vedic Mathematics to the mathematics teachers and kept them all absorbed.

This will signify how useful Vedic Mathematics can be in making mathematics a matter of joy. After his higher secondary examination is over, Manish has promised to go round towns and organize classes in Vedic Mathematics for all who are interested.

We have now an offer from the Panchmahals district for a course in Vedic Mathematics, another from Pilwai, Dr. A.K. Patel's College, and a third from Banaskantha.

If we get the necessary financial support from the Pratishthan, we feel confident, we can make Vedic Mathematics popular in Gujarat paving the way for its inclusion in the general mathematics curriculum.

Synthesis of Scientific Understanding of the Human Personality

ASHOK SHARMA

The unique distinguishing feature of man is the ability to integrate the vast amount of fragmented scientific knowledge into the understanding of the totality of human personality as a unified system.

Heisenberg's Uncertainty principle governs the uncertainty of naturally occurring inanimate objects whereas the human body behaviour is determined by stimulus response relationship with undefined uncertainty.

Therefore, the scientific delineation of human personality requires the understanding of the mechanism of stimulus response process in human body. The response of a man to a given stimulus can be divided into the following two categories:

1. Spontaneous or Instinctive Response.
2. Thoughtful or Conscious Response.

It is well-known that the instinctive component of human personality depends mainly on genetic factors and the conscious component of human personality is amenable to the effects of environment and training. Therefore, we shall make an attempt to scientifically understand the conscious component of man through the understanding of the information processing mechanism of human body through its brain and nervous system.

The human mind is an information processor. A careful study and analysis of the information processing sub-system of human body can lead to a more realistic understanding of human personality, which can be utilized to develop very efficient and effective techniques of human resource development.

The Objective of Human Personality is the Realization of True Knowledge

The objective of human personality is the realization of true knowledge. Briefly, it can be said that true knowledge can only be realized if the difference between the objective component of knowledge generated by intellectual component (बुद्धि) and the subjective component of knowledge

generated by the emotive component (मन) of the individual mind is eliminated.

It may be mentioned that the ancient Indian system of thought was a comprehensive system of the realization of true knowledge so as to attain a state of non-variable happiness. It is well-known that Yoga Darśana is based on behaviours of Response, Sāṅkhya on knowledge of invariable elements, Nyāya on evidence, and conventional science on cognition.

Table I gives the Five Techniques of Knowledge Generation:

Table I

Steps of Stimulus-response Process	*System of Generation of Knowledge*	*Technique of Generation of Knowledge*	*Objective of Generaion of Knowledge*
Perception of stimulus	Vaiśeṣika Darśana	Understanding of perception element	Realization of true knowledge of self
Cognition	Conventional	Objective component of cognition of several minds.	Understanding of the phenomenal world
Evidence	Nyāya Darśana	Understanding of direct, referential and intuitive evidences	Realization of true knowledge of self
Knowledge	Sāṅkhya Darśana	Enumeration of components of knowledge	Realization of true knowledge of self
Behaviour of response	Yoga Darśana	Modification of mental state of consciousness	Realization of non-variable and non-reactive state of mental consciousness

It can, therefore, be concluded that the ancient Indian system of thought comprehends the totality of knowledge as a classification of the cognized element of information of the conscious stimulus-response process of human personality rather than that of a phenomenal world separate from

man as adopted by conventional science. In view of the integrated systems view of human personality presented in this paper, one finds that the methodology of Vaiśeṣika Darśana is remarkably comprehensive and is an elegant analysis of cognized information elements generated by the information processing sub-system of human personality. The flow chart explains the viewpoint diagrammatically.

FLOW CHART

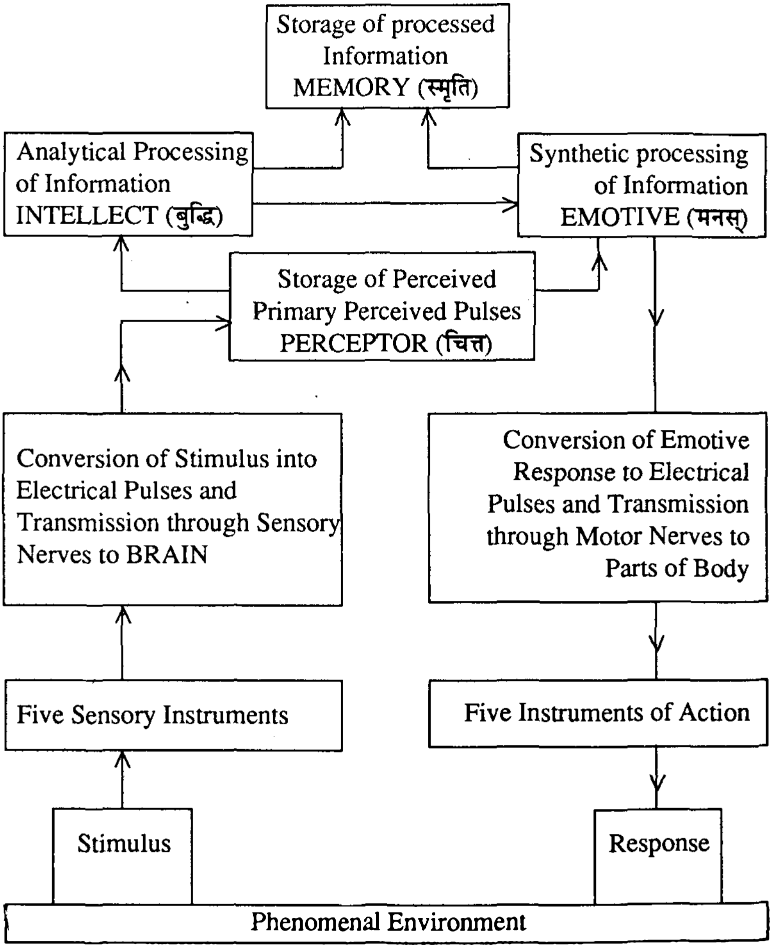


Figure 1

Leading Mathematicians of Ancient India

J.L. BANSAL

I. Vedic Period to First Century A.D.

The mathematical ability of the Vedic Hindus is reflected in the Saṃhitās, the Kalpasūtras, and the Vedāṅgas (Beginning 2500–2000 B.C., ending 750-500 B.C.)

Every household maintained three types of *Agnis* (fires), viz. *Dakṣiṇa Gārhapatya* and *Āhavanīya*, in altars of different special designs requiring the properties of triangles, rectangles, squares, etc. Several references to these are available in the *Ṛgveda-Saṃhitā*, the *Taittirīya-Saṃhitā* and the *Taittirīya Brāhmaṇa*.

In the handing down of instructions, the need for instructions in a written form was the origin of several *Śulba-sūtras*. Only seven of the *Śulba-sūtras*, after the names of the Ṛṣis, viz. (i) *Baudhāyana*, (ii) *Āpastamba*, (iii) *Kātyāyana*, (iv) *Mānava*, (v) *Maitrāyaṇa*, (vi) *Varāha*, and (vii) *Vādhūla* were written during 600 to 300 B.C. These are minor works and do not throw any additional light. In fact some of the *sūtras* cover roughly the first two books and the sixth book of Euclid (330-275 ? B.C.) Simple fractions, e.g. 3/8 (त्रि अष्टम), 2/7 (द्वि सप्तम), and the operations on them are available in the *Śulbas*. (1500 - 500 BC)

Vedic Hindus adopted 10 as the basis of numeration. The various recensions of the *Yajurveda-Saṃhitā* give names to the numbers as large as 10^{12}.

The Jainas regarded mathematics as an integral part of their religion. A section of their religious literature was named *Gaṇitānuyoga* meaning system of calculations. Amongst the religious works of the Jainas, featuring some kind of mathematics written during 500 B.C. to A.D. 100 are *Sūrya-Prajñapti*, *Jambudvīpa-prajñapti*, *Sthānāṅga Sūtra*, *Bhagavatī Sūtra*, *Uttarādhyayana Sūtra* and *Anuyogadvāra Sūtra*.

Āryabhaṭa I (b.A.D. 476)

In the history of Mathematics Āryabhaṭa I of Kusumpura, a native of Kerala, occupies a special position in Hindu mathematics. His mathemati-

cal rules are given in *Gītikapāda*. In his *Gītikapāda*, he introduces the alphabetical system and decimal numbers; tables of astronomical constants. Trigonometrical sine and other numerical data are given, along with rules for the extraction of square and cube roots, areas of triangles, trapezium, volume of pyramid and sphere, value of π, arithmetical progression, summation of series, interest, rule of three (rule of proportion), fractions, method of constructing sine tables, and indeterminate equations of the first degree.

Varāhamihira (c. A.D. 505)

In A.D. 505 Varāhamihira, son of Ādityadāsa of Kapitthaka wrote the famous *Pañcasiddhāntikā*, and *Bṛhat-Saṃhitā*. His most important contribution is the preparation of the corrected version of the Indian calendar with the warning for the future that the calendar should be periodically corrected by taking into account the accumulated precision.

Bhāskara I (c. A.D. 600)

Well-known for his *Mahābhāskarīya*, and a shorter tract *Laghubhāskarīya*, Bhāskara I primarily developed Āryabhaṭa's principles of astronomy. In mathematics, his main contribution lies in the field of indeterminate equations of the first degree. He found a method to solve such equations with two remainders.

Brahmagupta (b. A.D. 598)

After Varāhamihira the most celebrated mathematician appears to be Brahmagupta born in A.D. 598 who wrote his *Brahma-sphuṭa-siddhānta* in his thirtieth year. Brahmagupta called the 12th chapter as *Gaṇita* and the 18th chapter as *Kuṭṭaka* (algebra). The Abbasid Khalif al-Mansoor (A.D. 712-775) of Baghdad got *Brahma-sphuṭa-siddhānta* translated in Arabic in A.D. 770 with the help of Kaṅka of Ujjain. Brahmagupta also obtained the solution of the indeterminate equation $Nx^2 + 1 = y^2$.

Mahavīrācārya (c. A.D. 850)

Mahāvīrācārya, the most celebrated Jaina mathematician of the ninth century wrote *Gaṇita-sāra-saṅghraha* in A.D. 850, which contained mostly pure mathematics, giving operations with numbers, squaring and cubing,

square roots and cube roots, summation of arithmetic and geometric series, fractions, rule of three, mensuration and algebra.

Several astronomers who flourished between the time of Brahmagupta and Bhāskara II (A.D. 1114) are Vaṭeśvara (A.D. 880), Mañjulācārya (*c.* A.D. 932), Āryabhaṭa II (*c.* A.D. 950), Śrīdharācārya (A.D. 991), Śrīpati (*c.*A.D. 1000) and Śatānanda (c. A.D. 1100). Also Āryabhaṭa II (A.D. 950) is the author of *Mahāsiddhānta*, an astronomical compendium based on the orthodox tradition of the *Smṛtis.* Bhāskara II (*c.* A.D. 1114), the most well-known of the mathematicians of ancient India wrote his famous work *Siddhānta-Śiromaṇi* in the year A.D. 1150, divided into four parts, *Līlāvatī, Bījagaṇita, Golādhyāya,* and *Grahagaṇita.* A rough concept of infinity also occurs in *Līlavatī.* He was the first to concieve the differential calculus and gave $\sin y' - \sin y = (y' - y) \cos y$ which is equivalent to $\delta (\sin y) = (\cos y)\ \delta y$.

Then came the era of commentaries on *Līlāvatī: Gaṇita-kaumudī* (1350), *Ganitāmṛta-sāra* (1420), *Buddhi-Vilāsanī* (1540), *Gaṇitāmṛta* (1538), *Sūrya-prakāśa* (1541), and *Vāsanā-Bhāṣya* by Raṅganātha (1573).

The political upheavals roughly stopped the academic activity, save for Maharaja Sawai Jai Singh II of Jaipur (1686-1743), a distinguished soldier, statesman, politician, astronomer and mathematician. By his order Samrāṭ Jagannātha translated Euclid's *Elements* into Sanskrit under the title *Rekhāgaṇita* from the Arabic version *Tahrir-ul-uglidaṣ* by Nasir-al-Din of Persia. He also built five stone observatories, one each at Jaipur, Mathura, Varanasi, Delhi and Ujjain.

Then came the outstanding genius of S. Ramanujan born in 1887 whose works are being pursued with great zeal the world over during his birth centenary.

Structural Frames and Systems of Gaṇita Sūtras

S.K. KAPOOR

The main question about the source as well as potentialities of the *Gaṇita Sūtras* is whether the *Gaṇita Sūtras* admit structural frames and systems and if so whether those structural frames and systems are in unison with the structural frames and systems of basic Vedic alphabet format permitting operations in terms of the composition rules of the basic Vedic alphabet format. Consequently we start with the compositional split up of the *sūtras* and *upasūtras* and examine the Vedic claims enunciated below. We have developed a ten-point plan based on the following propositions:

a. The Vedas are Śrutis and not compositions.
b. The Vedas are written with rays of the sun.
c. The Vedas are a universal set of knowledge.
d. The Vedas are not according to grammar, rather grammar rules are derived from the Vedas.
e. All languages spoken by human beings are derivable from the Vedas.
f. Whenever man may process out while in interaction with the universe at any point of time, he shall find that that already stands processed in the Vedas.
g. The names of the objects, etc., of our universe are formulated in accordance with the Vedas.
h. The Vedas can be comprehended by meditation (deep contemplation) on *Ānahata Nāda*.
i. The Vedas can be comprehended by *Trāṭaka* on the sun.

Therefore,

1. The need of the present generation is to have a proper processing model to approach the Vedic claims.
2. Towards this, the celebrated work of Svāmī Bhāratī Kṛṣṇa Tīrthajī Mahārāja, entitled *Vedic Mathematics* revives our hopes that the Vedic claims can be scientifically, rather mathematically, approached.
3. To approach the Vedic claims would mean to approach the Vedic alphabet first of all. For this, the source material would be the inner evidence of the Vedic literature itself.

4. In Vedic literature the starting and ending point is OM (ॐ).
5. *Śrīmad-Bhagavad-Gītā* preserves for us the knowledge that the Brāhmaṇas (ब्राह्मण), the Vedas (वेद), as well as the sacrifices (यज्ञ) are contained within the threefold designation of the absolute, i.e. *OM* (ॐ), *Tat* (तत्) and *Sat* (सत्).
6. The *Praśna-upaniṣad* holds that anyone who meditates on the Supreme Puruṣa with the help of this very syllable OM (as AUM possessed *Tri-mātra*, three letters) becomes unified with the Sun consisting of light.
7. *Śrīmad-Bhagavad-Gītā* also states that Lord Krishna himself had imparted the Eternal Yoga to the Sun, who transmitted it to Manu. This flows as Eternal florescence, Divya-gaṅgā.
8. *Ādi Vāmīki Rāmāyaṇa* describes Divya-gaṅgā as :
   'The eternal florescence flows into the forehead of Lord Śiva from where it was channelized into Bindu Sarovar, from there the Florescence Process stood streamed as seven-layered flow which ultimately gets crystallized into three channels constituting a single processing process before it followed the chariot of king Bhagīratha on this Earth.'
9. All this is Vedic mathematics. The conditioning of a three-dimensional frame for the universe being provided mathematical structure, it would become mathematics of florescence systems of sunlight as a Divya-gaṅgā flowing through OM formulation.
10. To build up the conceptual base for a regular mathematics course, I give one illustration to give an idea of how it works (*Māṇḍūkyopaniṣad*): Brahman admits four quarters. As such the Source Alphabet is to admit four components only. That is why the OM formulation splits up as under :

1	2	3	4
	∪	3	7

and that is why the Vedas are four and four are the states of the Ātman.

	1	2	3	4
Brahman	First quarter	Second quarter	Third quarter	Fourth quarter
Om	.	∪	3	7
Ātman	Waking State	Dream State	Deep Sleep	Oneness
Vedas	Ṛgveda	Yajurveda	Sāmaveda	Atharvaveda

Propagation of Vedic Mathematics

DILEEP KULKARNI

In a science supplement of one of the newspapers, there appeared a cartoon. There is a small boy, a student, showing his notebook to his teacher, on which is written $2 \times 2 = 3$. He had got this answer from his personal computer. The teacher, after seeing this answer, catches the ear of this small boy and says, 'You fool! go and get your computer repaired. Here is mine which gives the correct answer : $2 \times 2 = 5$.'

It is very easy to laugh at this joke; just because we know that $2 \times 2 = 4$. But, suppose, we want to know how much is 79232×53873; and if two different calculators or computers gave the answers as 4729894556 and 4297600436, what would be our reaction? Can we laugh at the answers so easily?

It thus becomes important, even in the age of computers, to have some non-technological means to perform mathematical calculations.

Secondly, there should be as many methods as possible to perform calculations. While doing arithmetical calculations, we deal with ten numbers, 0 to 9. Each one of these has got peculiar properties and ways of behaviour. If these are closely looked at, we can have newer and newer methods of doing calculations.

Vedic Mathematics has got both these advantages. It is a good means of performing calculations mentally, it also keeps before us more than one way of doing a particular calculation. This thing is possible because Vedic maths looks at numbers from different angles. When 29 is looked at as 29, it has some properties. But as soon as we look at it as $30 - 1$ ($= 3\bar{1}$), we have an altogether new world opened up for calculations.

The second feature of Vedic maths is that most of the time it looks for patterns of numbers instead of individual digits. Instead of treating 998 as any three-digit number, it looks at the whole number and recognizes it as a number near a base. Once this property is recognized, easier methods of multiplication, division, etc. can be adopted.

One more feature which I find noteworthy is that in Vedic maths, instead of individual digits, attention is given to interrelations of digits. Multiplication and recurring decimals are good examples of this feature. After sufficient practice, this develops into an outlook which is becoming

more and more desirable these days. Our fragmented approach towards everything has created all sorts of problems. If through Vedic maths we can develop a holistic approach — start looking at the interrelatedness of everything — it will make our life better. Today, particle-Physics tells us about the interconnectedness of everything in the universe. Vedic maths does the same in its own way. This philosophical quality of Vedic maths will help develop a holistic, ecological approach towards life.

Because of these qualities — a special type of *modus operandi* we can say — Vedic maths has certain advantages. First, the calculations become easier. They can be done with less mental energy and in less time. But, at the same time there is full involvement. This is very important. Calculators do give answers in less time and with still less mental energy. But our involvement in the process of calculation is nil. This is highly dangerous in the long run, as we lose our ability to calculate. Vedic maths, on the contrary, helps us sharpen our calculating ability. Secondly, because of the simplicity and the availability of more than one method, the job of calculating becomes easy and interesting.

Because of all these advantages, teaching Vedic maths to school and college-going students will help a lot. I congratulate the Department of Human Resource for taking an initiative in this direction.

My colleague Shri B.G. Bapat and I are engaged in such an activity for the last seven or eight years. I would like to share with you our experiences in this regard. It was in 1981 that Dr. Andrew Nicholas was here in India. After his lecture in Bombay, there appeared an editorial in one of the newspapers in Pune. After reading that, many people were found to be interested in learning the subject. So, both of us decided to conduct a coaching class on Vedic maths. This was run on Sunday evenings. The batch of students was taught all the arithmetical calculations. We wrote an article about this class, as well as about the opinions of participants; and invited applications for a fresh batch. More than 250 people from the age of 9 to 70 enrolled. After that, a 10-day daily one hour class was conducted in the summer vacations of 1982.

After hearing and reading reports of these classes, peoples from various places started inviting us for conducting classes at their places. As that was not possible, we decided to serve the purpose through books. By writing and publishing one book every year (in Marathi), from 1982 to 1985, we have made available all the methods in the original book by H.H. Svāmī Bhāratī Kṛṣṇa Tīrtha to the Marathi knowing people. All the four books are selling well and have gone into more than one reprint. The total number of copies sold out since 1982 exceeds 20,000.

Shri Bapat and I have given lectures at many institutions. Shri Bapat is invited every year for teaching these methods to candidates appearing for the Bank Recruitment Examinations. Vedic maths is found to be highly useful for these, as well as scholarship examinations. I wrote a series of articles in Vivekananda Kendra's monthly, *Yuva Bharati*. Some of our students, in their turn, conducted similar classes. This will give an idea of the enthusiastic response received by us.

Based on this practical experience, I would like to put forward a multi-level programme for the propagation of Vedic maths.

1. *At the academic level*: Inclusion of Vedic maths in the curriculum right from the first standard. This will undoubtedly change the present attitude of students towards mathematics. They will find it an easy and interesting subject, as well as do the calculations knowingly, as against done mechanically now-a-days. In St. James Independent School in London, these methods have been taught for many years.
2. *At the research level*: Working further on the lead given by H.H. Swamiji, we should find our methods applicable in various fields of mathematics. Scholarships can be offered to persons interested in doing this type of research.
3. *Application in computers*: Application of Vedic maths methods in computers to increase their speed and efficiency. Experiments in this regard were done at BARC few years back. A method for finding our recurring decimals of 1/19 was tried out. The results were encouraging.

We hope, we will be able to chalk out in this workshop some programmes in all these three areas. It is high time that we did something.

It won't be out of place to mention a couple of questions asked by people every time we talk about Vedic maths. The first one is: 'In which Veda does this appear?' We should not feel guilty of telling them that Vedic maths does not appear in any of the Vedas. It has its roots in one of the *Śulba Sūtras*, and these being a part of Vedic literature, the subject is given the name 'Vedic Mathematics' by H.H. Swamiji. Moreover, all the aphorisms are constructed by Swamiji and methods given are also developed by him only.

The second question is about the fastness of the Vedic maths methods. One should be frank in telling people that all the methods are not equally

simple, neither are they always faster than calculators. This question will never arise if we emphasize one thing, i.e. Vedic maths is not just some tricks for doing arithmetical calculations. It goes far beyond this and becomes a science in itself, useful in solving algebraic equations in plain and solid geometry, trigonometry, etc., subjects coming under higher maths. Once this is made clear, comparison with the speed of calculators and computers will not be made of by people. Putting forward Vedic maths as a science, and not as a pack of tricks or magic thus becomes important.

There is a vast scope for research in the subject and with the co-ordinated effort of scholars in Vedic mathematics, Sanskrit, and computer technology, a lot can be achieved.

Vedas as Science: A Brief Introduction

K.C. KULISH

An entirely new interpretation of Vedic knowledge has come to light for the first time in history. The meaning of Vedic terminology is revealed in a way to prove that the Vedas are nothing but science and the Vedas cover all aspects of the universe.

It is an irony of the situation that such treasure of knowledge remained hidden for quite a long time. Two Sanskrit scholars who worked for about 70 years and wrote about one hundred thousand pages in Sanskrit and Hindi from the late nineteenth to the mid-twentieth century devoted their entire life to writing and studying, but could not bring their valuable work to light.

Pt. Madhusudan Ozha, the temporal teacher of the Maharaja of Jaipur in the last quarter of the last century, was the real finder of this knowledge who produced more than two hundred books till his death in 1939, but did not like to divert his attention to propagate his knowledge at the cost of writing and teaching;the only public activity on his part was to deliver lectures at Oxford and Cambridge Universities in 1902 and a few lectures in Varanasi (India). He visited London along with the Maharaja on the occasion of the Coronation of King Edward VII. Then he was invited by the Indologists of Cambridge and Oxford. His visit to two London Universities was reported in the *Westminster Gazette* and *The Sun*, where he was described as the 'Human. Storehouse of Vedic Knowledge'. Professor McDonald and his wife organized a reception in his honour in Oxford.

Pt. Madhusudan Ozha trained about twenty disciples to carry his scientific message further, but none except Pt. Motilal Shastri could do much. Pt. Motilal continued to work on the lines of his Guru and wrote more than fifty thousand pages to elaborate the knowledge of Ozha, in Hindi. The President of India, Dr. Rajendra Prasad, invited him to deliver five lectures on Vedic Science in 1956 before an audience of top-level scholars and citizens of Delhi. The President was so impressed by his depth of knowledge that he himself became a patron of the research institute founded by Pandit Motilal. Unfortunately, Pandit Motilal and Dr. Rajendra

Prasad both died in 1961 and 1962. Since then the work remains at a stand still. The works of both the scholars are lying neglected even now.

Modern science is very dependent on laboratories and is popularly identified with machines. Its methods are experience, observation, calculation, and analysis; but this is not enough. Science is knowledge which can be sometimes tested in laboratories and some times not. There are elements which cannot be reached by any material device. According to Vedic science Prāṇas are the elements which are beyond matter and material world. They cannot be seen, or touched, or heard, or smelt by our senses. They are the source of all movement or motion in the universe but in themselves they are abstract and beyond material things. They cannot be detected by any material instrument or device. The Vedas give very vivid and detailed knowledge of Prāṇas. They are the basis of every existence. For instance, take handful of sand, mix water with it and make a ball of sand. Keep the ball of sand for some time. The water will evaporate, but the ball will still remain in the same shape, in spite of the water having dried up. Here is the role of Prāṇas. The element of Prāṇa in the sandball cannot be detected by any laboratory. The Prāṇas existed in small particles of sand and drops of water too and they exist in the sandball as well. They exist everywhere, but in an abstract form.

Another example can be cited of Prajāpati. What is known as a central point can be termed as Prajāpati in the Vedic language. Everything has a centre or central point. Take a pencil and put it on your finger horizontally. When it strikes a balance it will cease to tilt on either side. Here lies the centre of the pencil. What the shape, size, colour, or nature of the centre is, cannot be seen or described. It can only be felt, but cannot be demonstrated in any concrete form. It cannot be analysed in laboratory, but it is the supreme element in every existence and it is termed Prajāpati in each object. It can even be declared as the supreme or absolute power which is popularly called God or Īśvara.

What I mean to say is that science need not unnecessarily be tied to the laboratory. Science is knowledge and not technology. Technology is a by-product of science. Today science is identified with technology or the machine. There are things beyond the reach of the machine. The machine has its limitation and it will keep on changing from time to time. A telescope built today is more powerful and superior to one that was built a hundred years back. With the advancement of telescopic technology our power of observation in sky was also enhanced and changed. In the everchanging technology, our preception of things will also keep on changing and we shall never be able to reach a firm conclusion.

The Vedas, therefore, are based on scientific theories produced by visionaries and tested and tried through Yajña. Yajñas are used as laboratories to test the theories but the fundamental nature of science is pure knowledge. The scientific principles of the Vedas did not develop through tests and trials, but emerged out of the supreme form of knowledge. Vedic principles or theories based on such knowledge do not change. For instance the Vedas say that nothing is indivisible and they stick to it. Modern science till recently believed that the atom was indivisible. During World War II, after splitting the atom, the theory was changed. The Vedas say that planets and stars do not change their position but many scientists say that they do change. They also say that the universe is expanding but the Vedas see the universe otherwise. The Vedas say that things change when they have certain limits. Expanding and shrinking are relative terms. Things expand when they have definite shapes, areas, limits, or boundaries. While talking of expansion of the universe, we have to first determine its limits, otherwise we have to understand that any matter or element expands within its own limits. Without determining or knowing the limits of anything we can not conclude that it is expanding or shrinking.

The Vedas are quite clear on their theory that the whole universe is governed by certain laws. The same laws govern the movements of stars and planets. The Vedic term for planet is *Graha* and for star is *Nakṣatra*. The meaning of *Graha* is firm grip and *Nakṣatra* is a thing which does not erode. From these it follows that planets and stars do not change their position at all. To say that stars are coming downwards is not correct. According to the Vedic definition, the planets move on their routes in well-set positions. For example, the moon moves round the earth on its route, known in the Vedas as *Kakṣa*. The earth moves around the Sun along the route known as *Krānti*, the Sun moves around the Parameṣṭha on the route named *Āyana* and Parameṣṭha moves round the Svāyambhuva on the route called *Aṇḍa*. The Svayambhū does not move. It exists in the form of space and is the base for all other spheres and planets. All are created within this.

Vedic theory also explains that each planet is linked with another and their movement is governed by certain laws. If the Sun is coming down, the Moon and the Earth are also bound to come down. If all are coming down, then question arises, where to go? They all move in a circular way, therefore the theory of stars coming down or going up is difficult to explain.

One scientist explained to me in the course of a conversation that there are many Suns in the galaxy. True, but they are all governed by the same

law. The law which governs our solar system, governs others too. One solar system cannot be treated in an isolated manner. If one expands, all will expand, if one moves down, others will also move, but where? Is there any scope for expanding? If so, what is that?

The Vedas provide knowledge in minutest details about the origin, the formation, and the existence of the universe and each aspect of it. It needs deeper study of the Vedas through their own terminology. Pandit Ozha has shown the way and written several books on the subject. They need the attention of scientists and other scholars. They can certainly benefit from the great treasure of scientific knowledge.

The biggest contribution of Pandit Madhusudan Ozha is to bring the Vedas close to the day-to-day world. He proved that the Vedas are absolutely relevant to present-day life if their knowledge is applied in a scientific manner. His approach to the Vedas was to reveal the meaning of Vedic terminology. His theory was that the meaning of Vedic terms cannot be understood through grammar or through other languages, because there was no knowledge prior to the Vedas. Therefore the meaning has got to be discovered within the Vedic terms and he did the job.

Another significant contribution of Ozhaji was that he brought the Vedas out of the books. He, for the first time, revealed that the Vedas are not mere books but the fundamental elements of the creation of the universe.

He proved that each and every thing known is shaped by the Vedas. A tiny pin and a big mountain are both made of the Vedas, meaning thereby that the Vedas are everywhere and everything is full of the Vedas in the form of elements.

He deals with the formation of universe at great length. How was the universe created? How did the stars and planets take shape? How did the earth and life on it develop? What is the real nature of sound, Sun, Moon, Earth, Water, Air and Sky? What are and how many are the fundamental elements of the world? What are the principles governing the creation, existence and evolution of life?

Pandit Motilal further developed the knowledge imparted by his Guru. He wrote large volumes of a new commentary on the Gītā, Upaniṣads, Brāhmaṇas and the original Vedas in thousands of pages in Hindi with a view to making popular the knowledge of science and changed the whole concept of science and the Vedas, both. He took science away from laboratories and established it as different from technology. He said, everything is created by one. How is everything created, how it changes forms and what process does it undergo? To know this is the nature of

science. In Vedic terminology the Yajña is the laboratory of Vedic science where each theory is applied. Yajña is not merely the burning of ghee and grain in fire but synchronizing and harmonizing the human body with the cosmic body. The Vedic principle is that the body is the replica or miniature form of the universe. Whatever exists in nautre, exists in the body too. This theory is presented in the Mantra यथाऽण्डे तथा पिण्डे (yathāṇḍe tathā piṇḍe).

Ozha and Motilal Shastri, while describing the evolution of the universe, present it in five parts. Our knowledge so far is confined to three parts known as the Sun, the Moon, and the Earth, besides the stars, planets and the sky.According to these two scholars, the Sun is in the middle of the universe. There is a sphere beyond the Sun known as Parameṣṭha.The Sun circles around Parameṣṭha and the magnitude of this sphere is depicted in a way as to compare the Sun with a bubble in the ocean. The Sun completes a full round of Parameṣṭha in 25,000 years. There is said to be one more sphere termed as Svayambhūva. The Parameṣṭha circles around this sphere and this is depicted as a static sphere containing the fundamental elements of the creation known as Prāṇas. Prāṇas are the real Ṛṣis. They are in an abstract form. They can neither be seen nor touched, nor heard, nor smelt. They are beyond the reach of our physical senses. Svayambhūva is described as the origin of universe. The Sun, the Moon, the Air, the Water, the Light, the Earth and all other parts of the universe are created by and after this by the interaction of various Prāṇas and matters. The whole process is laid down in the minutest detail in an orderly manner, as a scientific system ought to be. But they very emphatically state that life is confined to the solar system only and does not exist beyond it. The very ingrediants of life do not exist beyond solar system. The zone within which life exists is known as Saṃvatsara with the solar system. If the zodiac is divided into two poles, North and South, the Saṃvatsara falls 24 degrees North and 24 degrees South of equator of the zodiac. The North is presented as the source of Soma and the South as the source of Agni. Both the elements continuously move towards each other. The interaction of these two produce season, including the Monsoon. They create life and everything on earth. Both the parts of Saṃvatsara are described as male (South) and female (North) and both combined are taken as one full unit. Both are described as instruments of recreation. It is Saṃvatsara which makes human body the replica of the universe. The 24 ribs in our body represent the 24 degrees of one part of Saṃvatsara. Male and female, both combined, are of the order of 48 degrees and the spinal cord represents

the equator of the zodiac. Both become the instruments of producing new life.

Discussing the creation of new human beings Pandit Motilal, in his commentary on the *Śatapatha Brāhmaṇa* also states that cohabitation of male and female is not necessary to produce a baby. This knowledge is derived from the Vedas. He says, there are sperms of human seed in the male semen and the female blood known as Vṛṣa and Yoṣā. Unless these two sperms join no physical exercise can produce a baby. Pandit Motilal says that Yoṣā and Vṛṣa exist in cosmos and they provide the sperm of their nature to human bodies. If one possesses the knowledge of these two elements, one can produce life even without physical contact of male and female bodies.

The example of the human body is given on the ground that the human being is the most developed species of all and it is the true replica of the universe known as Brahmāṇḍa in Vedic language.

Pandit Motilal has written a book in four volumes on genetics. The title of the book is *Śrāddhavijñana* which deals with the human body as whole, including the inner self. The precision with which he has dealt with the subject is a rare treat. I have talked to many doctors and Ayurvedic teachers on the subject. They frankly admitted they did not come across such a precise knowledge of anatomy, physiology, pathology, nerves and other properties. This book deals with not only the origin and formation of life but also with the life after death, and the author proves his point with reasoning.

Ozhaji dwelt with the formation of atoms in his book *Brahma Vijñāna* and firmly declares that the atom was never indivisible. There is nothing invidivisible he says, except the one which creates everything. The atom is also divisible. He gives a few examples also of the variety of atoms and their interaction with Prāṇas and other atoms as a source of producing new matter and new forms.

While discussing the formation of stars and shapes of things, Ozhaji categorically rejects the theory of light years based on the movement of light. The theory is that the light of the stars takes thousands of years to reach the earth. On this Ozhaji advances his theory based on the Vedas. He says that the nucleus of the Sun, its body and the radius of light, all three are produced simultaneously; they exist simultaneously and permanently. This theory is applicable to all stars and visible objects. He says that unless the last point of sun's ray reaches the earth, the very body of the sun is not formed. The Sun consists of the three Vedas, as all other things do. The

body of the Sun is Ṛgveda, the contents of the body is Yujurveda and the radius of light is Sāmaveda. All the three Vedas co-exist in every thing. They are never without each other. They are born together and they live together. To cite an example, we can see a paperweight. The solid mass of the peperweight is Ṛgveda, its nucleus is Yajurveda and its sphere of visibility is Sāmveda. The last point of the sphere of visibility and the body of the paperweight appear simultaneously. There is no gap and there cannot be any gap between the two the same theory is applicable to the light of the Sun.

No doubt the theory of 'light years' is based on certain observations and calculations, but not on the basis of any fundamental principle. The scientists have to give serious throught to the theory advanced by Ozhaji.

The theory of the light year is a recent one. In the same way the theory of the divisibility of atoms is also a recent one, but the Vedas have laid down theories on every big and small aspect of the universe and for the first time the two scholars of Jaipur have revealed the meaning of the Vedic terms in a scientific manner. Never before in history, have such studies been made. Whatever we see in the name of the Vedas is ritualistic commentary without real knowledge or stereotyped translations based on grammar. Pandit Madhusudan Ozha and his disciple Pandit Motilal Shastri, devoting their whole life, have presented the Vedas as pure science.

Pandit Motilal has written a 1000-page volume on our rituals, festivals and holy ceremonies to prove that the whole life-style of the Aryans and the society created by them is based on science. He has also dwelt on the subject of the Purāṇas which is known in the West as mythology. He revealed the meaning of symbols of various Gods and Goddesses and their mounts and vehicles like birds and animals. He declares that there is nothing like 'myth' in the Purāṇas.

Both the scholars have written in details on nature, water, air, fire, light, sound, comets and stars. They have prepared diagrams too to illustrate their knowledge. They do not talk in vague language. Everything is precise and accurate. There is no presumption, no guess work and no speculation. When they quote figures, they quote with precision and not in a roundabout way. For instance, they say that the average size of human body is equal to 84 fingers and they give a scientific explanation for it. They say that the genes are 84, two-thirds negative and one-third positive. There are 84 lakh species on the earth. They classifield the groups of nerves and veins in our body in various categories and counted their numbers. The same type of

knowledge has been revealed by them about space and cosmology, about planets and stars, about comets, about gases, about electricity.

One important aspect of their approach to science is that they did not ignore modern science. They made a comparative study of the various disciplines in science and had a basic knowledge of established scientific theories of modern times. Ozhaji even wrote small books as *Science Pradeep, Vastu Sameeksha, Padartha Vidya*, etc., based on modern physics and chemistry. Pandit Motilal also referred to all the great western philosophers, and scientific thinkers. He dwelt on the modern theories of the origin of the world and gave his explanations based on the Vedas. He was aware of the role of hydrogen. He touched on the Big Bang theory in his own way while dealing with Anāhata Nāda which he presented as the origin of all sounds and all matter. Pandit Ozha has written a large book on weather science, touching on many unknown aspects of the subject.

A former Deputy Director General of MeteorologyDepartment and fellow of the National Academy of Science, Dr. Ramanathan, has written ten papers on the basis of Ozhaji's writing.

It is very difficult, rather impossible, to detail all the work done by these two great scholars. This is not even a short glimpse of its magnitude and depth. They need attention of the whole scientific community, states and institutions of knowledge, to make use of and bring this knowledge to light for the benefit of mankind. This is knowledge cutting across all barriers of race, religion, state and political systems. This is for the good of all and against none.

Vedic Mathematics : Some Ramifications

ABHAY KASHYAP

Vedic Mathematics, or VM as it is referred to, has been generating considerable interest among various circles that look forward to some advantage that may accrue from VM towards the attainment of their goals and objectives.

It has, therefore, become necessary to examine with an open mind that what, the why and the how of VM. Only then shall we be able to arrive at a strategy which develops the potential of VM to its fullest.

What is Vedic Mathematics ?

Is it some, as yet unknown to most, complete-in-itself mathematics which needs no further additions to it? Is it, in other words, complete knowledge? Perhaps it is, but I am sure that even the most ardent supporters of VM will not take this stand.They will at least concede that even if VM was once complete, considerable research has to be done to bring it back to its original form.

As of today what I understand to be VM is represented by the 16 *sūtras* as presented by the Jagadguru. It is these that are being discussed and researched as VM. However, I propose a broader definition of VM.

VM is the study of mathematical relationships in keeping with the Vedic tradition of intuitive thinking.

In other words, Vedic Mathematics is an approach which exploits both halves of the brain by using the pattern recognition capabilities of one and the analytical capabilities of the other.

This definition does not preclude further work on the subejct and also takes into account the aspect of 'Darśana' which is inspired pattern recognition through which these sixteen *sūtras* have reached us. This definition also implies a secular meaning of VM, i.e. Vedic — that which is spoken. This must not be taken to have any pre-conceived religious overtones.

It is very important here to clarify what VM is not.

VM in its present form is far from complete mathematics. It must also be clarified that VM has little to do with artificial intelligence or NASA 20 year programme. The bulk of research being conducted in Europe, USA and even the USSR on Sanskrit is in the area of Sanskrit syntax and grammar. Here we must not forget that they are as much inheritors of the Sanskrit tradition as we are, in the sense that their languages belong to the family of languages derived from Sanskrit. The reason they are examining Sanskrit is to develop an interface between computer understandable languages and natural languages. This must not be confused with VM.

Perhaps it would be useful for the sake of perspective to clarify that there are three things related to Sanskrit which have an inherent advantage over other languages:

(1) Structure
(2) Script — Devanagari
(3) VM.

In terms of potential for providing a technological window to leap into the forefront of technology VM is the least important. This is in no way to undermine the importance of VM, but only to show that Sanskrit or Vedic literature has other things to offer of even greater potential.

In scope Vedic Mathematics, as presently understood, is broadly the same as the school-leaving level. Further work needs to be done to broaden the scope of VM.

Why VM?

The question as to why we should use VM can only be answered effectively by first examining the potential of VM at different levels of mathematical sophistication. This can only be done on the basis of identifying the needs in those segments, these may be active or dormant, and these could be addressed by VM.

In other words, if we want VM to be introduced effectively to the population in need of mathematical literacy we must focus our efforts, avoid over-generalizations and desist from glossing over the failings of VM or of our own understanding of it. I feel that the following needs are felt by the different levels of mathematics users:

1. Students feel maths is cumbersome and difficult at the school level.
2. Persons with an arts background have to appear for competitive exams., e.g. M.B.A., Banks, etc., where the lack of mathematical skills often proves to be their undoing.

3. An analytical capacity is needed to inculcate a proper scientific spirit to study sciences at the graduate and the post-graduate levels. The methodology is perhaps sometimes buried under cumbersome procedures and calculations eating into valuable time needed for the study of concepts.
4. Due to the convergence of technologies, information technologies (IT) have become the key to national economic development and the strength of a nation is now measured in terms of information technology indicators. The key to IT is computers. The know how for these is closely held by the existing technologies. However, new techniques are emerging which, if developed, can help a technological jump. Can VM help in this?

As far as the first need is concerned I feel that the advantage that will accrue may be limited. However, as it is difficult to substantiate my assessment in the light of enthusiastic reports on VM shall only state my reasoning. I feel that the capacity of a child to learn is not stretched to its limits by the present maths syllabus and therefore even if some calculations become more intuitive it will be of little immediate advantage.

However, if this makes the inculcation of intuitive thinking possible in an entire generation it will result in considerable long-term benefits.

The second need is very real and immediate and simple multiplication and division *sūtras* are all that are needed to help these students. However, it is not clear as yet what competitive advantage will these candidates have when everyone has access to these *sūtras*.

The third requirement of the graduate and post-graduate levels is true of places where the use of calculators and computers is not allowed, as otherwise there is no saving in time. However, further development of VM may result in solutions to problems which may propel developments in unexplored areas.

The need in the field of information technology and computers, if addressed, will have far-reaching implications for our national development, both economic as well as political.

Let us examine if VM can help us at all in this area.

It is well-known that for computers to work, all problems are to be reduced to the four basic functions +, –, ×, ÷. Even these are actually implemented through only two functions, × and +. Division remains a problematic issue for computers handled in a roundabout way. With the above constraints it is relevant only to see if VM can improve computer performance in the case of any of these operations. Promises of better

performance at higher levels will have only marginal benefits as they will essentially be better software algorithms which will give better performance on existing computers.

Let us take the case of multiplication. What is important here is to examine the number of operations in the conventional method and in VM. On close examination these will be found to be same; in which case it becomes clear that in existing computer architecture in which one process is done after the other, no advantage will accrue. On the other hand let us examine the new parallel processing computer architecture in which several computers break up the problem into smaller ones and process them simultaneously.

Here again closer examination shows us that Vedic as well as our present maths can be broken up into precisely the same number of independent operations. In fact VM requires additional effort to keep track of placement of numbers in the resultant answer.

Therefore, to existing as well as emerging architecture VM actually is a disdvantage. Does that mean VM is of no use here?

I am now taking the liberty of making a proposal which has far-reaching implications and to the best of my knowledge has not been made as yet.

A new type of computer may be designed which has a long, i.e. several bits wide, register for answers but has a short register for processing parts of the problem. The position of digits in the answer can be prefixed using the symmetry of numbers, as done manually.

Such an effort will cost a considerable amount of money but will enable us in India to beat the U.S. and Japan in AI and pattern recognition, which is essentially a very number-intensive process. With the proposed structure a very high speed in number processing can be achieved. This, coupled with the application of Sanskrit syntax and Devanagari script can bring India to the threshold of an educational revolution for which literacy will be redundant.

In conclusion I would like to point out that we have a long way to go and it is most important that we take up the opportunity offered by this workshop to define our focus so that our energies are not dissipated.

How VM?

As far as the introduction of VM at school level is concerned it can be brought about slowly by increasing the level of recognition in the general environment of VM, and then introducing courses at appropriate levels of

VM. It must be kept in mind that a considerable change in the outlook of teachers will be called for. This means a considerable effort in teaching teachers.

As a first step, the examination boards of different states and the centre may allow the solution of examination papers using Vedic Mathematics. The student must be given the assurance that the use of Vedic Mathematics will not be penalized; and only then will the student try to exploit the advantage offered by VM.

For encouraging the use of VM by students appearing in exams an attempt must be made to introduce them toVM through the so-called training schools for such exams, besides reaching them by multi-media effort to spread VM.

At the present stage of development further research in VM should be encouraged at different universities by giving active encouragement to students who want to use and enquire into VM. Much must be done to avoid a narrow definition of VM and its scope must be kept as broad as is in keeping with the Vedic spirit.

Its application in the field of computers can be taken up as a significant time-bound project, fully backed by the government. If this idea is found interesting, the Centre for Emerging Technologies could take up the work of preparing a project report for the same.

In general, I would like to emphasize that we msut try to inculcate the spirit of VM.

Vedic Mathematics : An Appraisal from the Perspective of Modern Mathematics

S.N. PANDEY

Mathematics can be considered from two major viewpoints. The first is to use mathematics as tool in everyday life for solving our practical problems. Here our main concern is 'how' we solve our problems by performing certain operations on numbers or symbols. In the Vedas the counting numbers do appear in different contexts and certain numbers play very significant roles in the development of philosophical expositions. Even the word *saṅkhyā* means *samyag jñāna.* Knowledge gains perfection and unambiguity and clarity if it is expressed in terms of numbers. This is also the view about mathematics and this is why mathematics has been able to enter domains of bio-sciences, psychology, social sciences, etc. Not only this, the ever-increasing use of computers has made mathematics and numbers omnipresent. The Vedas are the oldest storehouse of knowledge on earth. This fact is universally accepted. The positional notation for expressing counting numbers with zero as a unique spacer is present in the *mantras* appearing in the Vedas and all historians agree that this positional notation of expressing natural numbers reached the western countries from India through the Arab countries. This Indian invention of expressing counting numbers has played a crucial role in the evolution and growth of modern mathematics and credit goes to the western mathematicians for this tremendous and amazing development of mathematical thought.

Then if I say that all mathematics is Vedic in origin, I am justified in my assertion. According to the details available today, if I say that the Vedas tell me how to solve a differential equation, I may be wrong. Again, if I say that everything that modern mathematicians are doing can be directly extracted from the Vedas, I may be wrong. Mathematics is a creative science like music, poetry, and painting.

Gaṇita is regarded as a Vedāṅga śāstra in our tradition and it is considered to be the 'eye' (नेत्रम्) of other śāstras. This is exactly the modern view when mathematics is used as a tool for constructing a model of a physical reality, a socio-economic system, a psychological or mechanical system.

यथा शिखा मयूराणां नागानां मणयो यथा ।
तद्वद् वेदाङ्गशास्त्राणाम् गणितं मूर्धनि स्थितम् ॥

There is no difference in the Vedic outlook and the modern outlook. The difference lies in the circumstances. Western thought was concentrated on the better understanding of nature and its working and more than that on the possibility of harnessing the forces of nature to serve human needs. Mathematics again became a tool, but a very sophisticated tool.

The Sanskrit grammar of Pāṇini, the system of Patañjali's Yoga, the Sāṅkhya system of grasping the evolution of the physical reality, to quote a few, are the marvellous achievements using an amazing combination of analytical skill, philosophical penetration, and interaction of the Indian ṛṣis. They are not found in the Vedas in the form they have been presented.

Similarly, a system of sixteen *sūtras* under the title *Vedic Mathematics* seem to have been restructured by Svāmī Śrī Bhāratī Kṛṣṇa Tīrthajī. In the course of evolution of the system more *sūtras* could be added. They are not final. They do not seem to embrace all areas of mathematics. It would be much appreciated if our Sanskrit scholars, trained in mathematics, concentrate on these *sūtras* in collaboration with professional mathematicians and extend the work of Swamiji further. But one thing is demanded from both the sides. The traditional Sanskrit scholars and pandits should realize the fact that the contribution of the West to knowledge is as 'pious' as the Vedas and the western-minded scholars and mathematicians should realize that the knowledge hidden in the Vedas and the śāstras is not 'trash'. It is as scientific and as systematic as any knowledge can be.

Scholars trained in the western tradition should also realize that 'history is philosophy' as it was pointed out by Karl Marx. When the colonial rulers ruled India, their motive was to destroy the cultural fabric of India by presenting a distorted history. One of our scholars, Dr. P.V. Vartak from Poona, has examined the astronomical data available in the *Mahābhārata* and he has come to the conclusion that the battle of Mahābhārata started on 17 October 5561 B.C. This, if accepted, demands a drastic change in the entire historicity developed by western scholars regarding the Vedas, the Purāṇas and the śāstras.

Let us look at Vedic Mathematics from this perspective and setting all controversies aside, work for the growth and development of the novel techniques of Vedic Mathematics.

Glimpse of Mathematical Heritage of Ancient India and its Transmission to other Countries

S.A. PARAMHANS

The richness of the cultural tradition of a country or of a race leans heavily upon the development of science and technology in the country and this development depends upon the treasure of mathematical knowledge and its applications in various spheres of public and personal life. Hence we can safely say that in order to measure the development of a race or a country during a certain period, its development in mathematics can be taken as a good indicator.

Thus to ascertain the past culture and development of India it is essential to examine the contributions of the savants of this country towards mathematics.

Arithmetic and Counting Power

In this context first we should notice that the natural numbers which the world knows today, are the boon of the Indian sages.[1] The striking fact to note here is that the Hindu genius gave it perfection at the very beginning. This is why G. Sarton, while mentioning the invention and development of spectacles in Italy, has remarked about its completeness:

> 'This is not a matter like Hindu numerals which were almost perfect from the moment of their creation.'[2]

Ginsburg describes how the numerals reached the West:

> 'The Hindu notation was carried to Arabia about 770 A.D. by a Hindu scholar named Kaṅka who was invited from Ujjain to the famous court of Baghdad by the Abbaside Khalif Al-Mansur. Kaṅka taught Hindu astronomy and mathematics to the Arabian scholars; and with his help, they translated into Arabic the *Brahma-Sphuṭa-Siddhānta* of Brahmagupta. The recent discovery by the French savant M.F. Nau proves that the Hindu numerals were well-known and much appreciated in Syria about the middle of the 7th century A.D.'[3]

The journey of the numerals is further described by Datta:

> 'From Arabia, the numerals slowly marched towards the West through Egypt and northern Arabia; and they finally entered Europe in the

11th Century. The Europeans called them the Arabic notations, because they received them from the Arabs. But the Arabs themselves, Eastern as well as the Western, have unanimously called them the Hindu figures (*Al-Arquan-Al-Hindu*).'[4]

Not only numerals, but also zero, one of the most important tools in mathematics, was a gift by Indian savants as is clear from Datta and Singh[5] and Halsted.[6] It was used as a number in India in the remote past. Otherwise, how could Indians give the system of decimal place value, on the basis of which, big numbers of the *Taittirīya-Saṃhitā* and the *Vālmīkīya Rāmāyaṇa* could be constructed? It is worth mentioning here that the *Taittirīya-Saṃhitā*[7] (before 1000 B.C.) gives terminology for the numbers of order 10^{19} as:

(Eka)	1	(Madhya)	10^{10}
(Daśa)	10	(Anta)	10^{11}
(Śata)	10^2	(Parārdha)	10^{12}
(Sahasra)	10^3	(Uṣas)	10^{13}
(Ayuta)	10^4	(Vyuṣita)	10^{14}
(Niyuta)	10^5	(Deśyata)	10^{15}
(Prayuta)	10^6	(Udyata)	10^{16}
(Arbuda)	10^7	(Udita)	10^{17}
(Nyarbuda)	10^8	(Svarga)	10^{18}
(Samudra)	10^9	(Loka)	10^{19}

As is indicated by the present author, the terminology for the numbers in the range $10^9 - 10^{19}$ varied slightly in the medieval period.[8] Not only this, the *Vālmīkīya Rāmāyaṇa*[9] has given the terminology for numbers of the order 10^{60} as follows :

'*Śatam* śatasahasrāṇām koṭimāhurmaniṣiṇaḥ' etc.

Thus :

$100 \times 1000 \times 100 = 10^7 = 1$ Koṭi
10^5 Koṭi $= 10^5 \times 10^7 = 10^{12} =$ Śaṅku
10^5 Śaṅku $= 10^{17} = 1$ Mahāśaṅku
10^5 Mahāśaṅku $= 10^{22} = 1$ Vṛnda
10^5 Vṛnda $= 10^{27} = 1$ Mahāvṛnda
10^5 Mahāvṛnda $= 10^{32} = 1$ Padma
10^5 Padma $= 10^{37} = 1$ Mahāpadma
10^5 Mahāpadma $= 10^{42} = 1$ Kharva
10^5 Kharva $= 10^{47} = 1$ Mahākharva

10^5 Mahākharva = 10^{50} = 1 Sumudra
10^5 Samudra = 10^{55} = 1 Ogha
10^5 Ogha = 10^{60} = 1 Mahaugha

It is to be remarked here that 'Samudra' which was used for 10^9 in the *Taittirīya-Saṃhitā*, has been used here for 10^{50} and all other terms for large numbers in this sequence are different from those found in the *Taittirīya-Saṃhitā*. It is noticeable that except this context, the numerical terms found at other places of the *Vālmikīya Rāmāyaṇa*, are from the *Taittirīya-Saṃhitā*. with the same meaning, e.g. Niyuta and Nyarbuda [10], Arbuda, Madhya, Antya, Samudra and Parārdha.[11]

It seems that the above terminology of the *Vālmikīya Rāmāyaṇa* for large numbers was not common, and was confined to experts who were highly specialized in the counting process since the numbers above Koṭi, as has been stated, were hardly used at any other place of the *Valmikīya Rāmāyaṇa* itself. This is why Śuka and Sāraṇa had first to define the terminology and then they gave the account of Rāma's army.[12]

Probably, due to the very limited use of the terms of the above sequence of large numbers, some of the terms disappeared in later literature, and those which are found, do not carry the same sense as above: the *Samarāṅgaṇasūtradhāra*[13] (11th century A.D.) uses Vṛnda for 10^9, Kharva for 10^{10}, Śaṅku for 10^{12} and Padma for 10^{13}.

Some scholars like David Pingree hold the view that 'the presence of names of multiples of 10 in the *Taittirīya-Saṃhitā* does not mean that the authors of that text used a zero.' It amounts to saying that 'one can form multiples of 10 without any familiarity with zero'. Anybody with an elementary knowledge of arithmetic can see the fallacy of the above assertion of these people. In fact, the use of multiples of 10 implies the knowledge of zero.

Also these people assert that 'the *Rāmāyaṇa* passages are late and also prove nothing about the use of zero in India.'

It is customary with these people to reject the passages they don't like by saying that 'these [passages] are later insertions.' Suppose this is the fact. Then Vālmīki could have given the counting of Rāma's army without using the numerical terminology in these passages. Had it been the actual situation, Vālmīki would not have concluded in the earlier counting as 'with their forces of incalculable number'.[14]

Also it is ludicrous to assert that a passage giving terminology in terms of multiples of 10^5, proves nothing about the use of zero in India. It is not

possible to imagine the systematic multiples of 10^5 without any knowledge of zero.

It is also curious that they should assert that 'the earliest actually corroborated reference to zero in India is in the *Yavanajātaka* composed by Sphujidhvaja in A.D. 269/270.' One should notice in this context that even ignoring the passages of the *Taittirīya-Saṃhitā*. and the *Vālmīkīya Rāmāyaṇa*, the use of zero is found in the *Chandaḥsūtras* of Piṅgala, approx. 200 B.C. (Datta and Singh, pp. 70-71).

Also these scholars assume that 'the zero had been used in Babylonian mathematics and astronomy and in Greek astronomy, long before this [i.e. A.D. 269/270]' for which they have no firm documentary proof.

The Buddhist treatise *Lalitavistara* (1st century B.C.) has given the terminology for 10^{53} as *Tallakṣaṇā*. The Pali grammarian Kāccāyana has provided the terminology for 10^{140} as *Asaṅkheyya*.

Some other historians of mathematics, e.g. Crossley, who are ignorant of these achievements of the ancient Hindus, have wrongly attributed to some later scholar (Nicholas Chuquet, A.D. 1494) the giving of terminology for large numbers (like 10^{12} and 10^{18}).[15] Albiruni, however, says: 'In counting who go beyond thousand, are only the Hindus.[16] Quādi-Sa'id also attributes to Indians the credit for developing numerical sciences:

> 'In the domain of numerical sciences, we have their [i.e., of Indians'] 'hisāb al-ghubār' which was explained by al-Khwārizmī. It is a very compendious and quick system of calculation, easy to understand, simple to adopt, and remarkable in its composition, bearing testimony to the sharp intelligence, creative power and remarkable faculty of invention of the Indians.'[17]

Really it is a remarkable mathematical invention of human race. The French mathematician Laplace remarks :

> 'It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position, as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit, but its very simplicity, the great ease which it has lent to all computations, puts our arithmetic in the first rank of useful inventions; and we shall appreciate the grandeur of this achievement when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity.'[18]

Diophantone Equations

The contribution of Indians, particularly of Bhāskara II on diophantone equations needs special mention. Crossley (p.193) quotes Bombelli (1572):

> 'But in recent years a Greek work in this discipline was rediscovered in the Library of Nostro Signore in the Vatican written by a certain Diophantus of Alexandria, a Greek author, who lived in the time of Antorius Pius, and it having been shown to me by Antonio Maria Pazzi Reggiano, Professor of Mathematics at Rome, and having judged with him the author to be well informed about number (although he did not treat irrational numbers, but in it there appears only a complete means of proceedings), he and I, in order to enrich the world with a work so finely made, decided to translate it and we have translated five of the books (there being seven in all); the remainder we were not able to finish because of pressure of work on one or other. In the work [we have translated] we have discovered much that [the author] on several occasions quotes from Indian authors, from which I learnt that this discipline was known to the Indians before the Arabs.'[19]

Applications

The major point to mention here is the importance given by Indian scholars to mathematical values and mathematical calculations. Mahāvīrācārya (A.D. 850) says in his *Gaṇita-sāra-saṅgraha* (Chapter I, Ślokas 9-19):

> 'In all transactions which relate to worldly, Vedic or other similar religious affairs, calculation is of use. In the science of wealth, in music and in drama, in the art of cooking, in medicine, in architecture, in prosody, in peotics and poetry, in logic and grammar, and such other things, and in relation to all that constitutes the peculiar value of the arts, the science of calculation (*Gaṇita*) is held in high esteem. In relation to the movement of heavenly bodies, in connection with eclipses and conjunctions of planets and in connection with *tripraśna* (direction, position, and time) and the course of the moon—indeed in all these it is utilized. The number, the diameter and the perimeter of islands, oceans and mountains; the dimensions of the rows of habitations and halls belonging to the inhabitants of the world, of the inter-space between the worlds of the world of light, of the world of gods and of the denizens of hell and other miscellaneous measurements of all sorts—all these are made out with the help of *Gaṇita*. The configuration of

living beings therein, the length of their lives, their eight attributes, and other similar things, their progress and other such things, their staying together, etc., all these are dependent upon *Gaṇita* [for their due comprehension]. What is the good of saying much? Whatever there is in all the three worlds, which are possessed of moving and non-moving beings, cannot exist as apart from *Gaṇita* [measurement and claculation].[20]

Therefore it is said:

यथा शिखा मयूराणां नागानां मणयो यथा ।
तद्वद् वेदाङ्गशास्त्राणां गणितं मूर्धनि स्थितम् ॥'

(Śloka 4)[21]

This subject was studied in this country in the remote past as we see in the *Chāndogyopaniṣad* (VII. 1.2.4) in the following event: Once upon a time Nārada approached the sage Sanatkumāra and begged of him the *Brahmavidyā* or the Supreme knowledge. Sanatkumāra asked Nārada to name the sciences and arts he had already studied so that he (Sanatkumāra) might judge what still remained to be learnt by him. There upon Nārada enumerated the various sciences and arts studied by him. This list included astronomy (*Nakṣatra-vidyā*) and arithmetic (*Rāśi-vidyā*). Thus mathematical science was not considered to be a hindrance to spiritual knowledge. In fact *Aparā vidyā* (secular knowledge) was then considered to be a helpful adjunct to *Parā vidyā* (spiritual knowledge).[22]

Rather, some eminent scholars, while commenting upon the *Yoga-sūtra* (I.39): "यथाभिमतं ध्यानाद्वा" which occurs in the enumeration (due to Patañjali) of the means of concentration, have said that mathematics is extremely helpful for the practice of concentration. Perhaps, because of this very reason one of the four branches of religious literature of the Jainas is *Gaṇitānuyoga* (the exposition of the principles of mathematics). The knowledge of *saṅkhyāna* (literally, 'the science of numbers', meaning arithmetic and astronomy) is stated to be one of the principal accomplishments of the Jaina priests. In Buddhist literature, too, arithmetic (*gaṇanā saṅkhyāya*) is regarded as the first and the noblest of the arts. All these give a fair idea of the importance and value set upon the culture of *gaṇita* in ancient India.

If we analyse and consider carefully the above statement of Mahāvīrācārya, we are sure to get difinite clues of applicatinos of mathematics. His statement, 'Mathematics is utilized in relation to the movements of heavenly bodies, eclipses, conjunctions of planets and *tripraśna* (direction,

position, and time)' makes it transparent that before the ninth century, Indian scholars were experts in computations regarding the movements of heavenly bodies and planets.

It is really amazing that Mahāvīrācārya talks about the computation of the diameter and perimeter of islands and mountains when there was nothing like so-called modern science. Not only this, he talks of the inner-space between the worlds when modern telescopes and rockets were quite distant prospects. The other things mentioned by him are still beyond the reach of modern science, as he talks about the dimentions of the world of gods and the denizens of hell (which are, in fact, beyond the belief of modern scientists). The calculation of the length of lives of living beings, is not unbelievable but is beyond the reach of modern science.

Thus *Gaṇita-sāra-saṅgraha* is enough to tell us about the development of science in ancient times in our country. It is really not unbelievable that the mention of *vimānas* in Indian scriptures initiated speculations about modern aeroplanes and rockets. In the words of Wingfield Petrie, Erich Von Daniken quotes from the *Mahābhārata* and the Purāṇas in support to his presumption of 'Ancient Astronauts'.[23]

Mahāvīrācārya's statements cannot be rated as mythical talk. The thirty-first chapter of the *Samarāṅgaṇasūtradhāra* of Bhoja (A.D. 1055) explains the technology of the aeroplane. Thus the accounts of space flights in Indian scriptures are not merely legendary but scientific. They can be traced in the *Rāmāyaṇa*, the *Mahābhārata* and so many other scriptures.

Astronomy

It is worth mentioning here that Indian Astronomy is the oldest of all the sciences of the human race. A few lines or pages regarding its development in ancient India will be like a drop in ocean. The effect of astral motions and the meteorology of the *Bṛhatsamhitā* are enough examples of the high achievements. Take for example, the studies on Jupiter. A few years ago ordinary people, and even modern scientists, had the opinion that Jupiter does not affect the earth (or the inhabitants of the earth) since it is at a very long distance. But modern scientific discoveries noted recently that it affects living creatures as well as non-living substances on earth. Let us praise our ancient sages who announced thousands of years ago the effects of the movement of Jupiter. According to them, Jupiter is responsible for the progress of living beings on earth (Chapter 8).[24] It affects them not only in mass but also individually. According to the Śāstras it is responsible for

the good attributes in man. Due to these reasons, our sages called it *Jīva*, (8.26; 17.18-19) and *Guru* (or Devaguru); (8.17; 17.26). Its name *Guru* is well-known to everybody in India. One can also trace its account even in the *Ṛgveda* (4.50.5): "बृहस्पतिः प्रथमं जायमानः"।

Gnomonics, the oldest device to measure time, is a great gift of Indian astronomy to the human race. Gnomon was used to find out seasons and some parameters of astronomical use, as is clear from the *Sūrya Prajñapti* (a Jaina canonical text on lunisolar kinematics). The device can still be seen in the Manamandir Observatory of Varanasi. Gaṇeśa Daivajña (A.D. 1522), the Author of the *Grahalāghava*, described this device in *Pratoda Yantra*.[25]

About the astronomical achievements of ancient Indian savants, the *Encyclopaedia Britanica* (p.518) remarks: 'The science for which the Brahmins, however, were most remarkable, is that of astronomy and in this their progress was so great as even yet to furnish matter of admiration to the moderns.'[26]

It should not be forgotten here that Indian meteorology could offer devices to forecast the weather six months in advance. Modern science has yet to take lessons from it. These astronomical achievements were based on mathematics.

Algebraic Equations

The construction of altars for *Yajñas* involved the problems of various branches of mathematics. So the algebraic equations date back to the *Śulba* period (before 500 B.C.). The transformation of square altars to rectangular forms involved the equation

$$ax = c^2.$$

Similarly in transforming the rectangular altars to square ones, the ancient Hindus had to consider the equation

$$x^2 + y^2 = z^2.$$

Example of integral solutions of these equations are found in the *Śulba-sūtras* as :

$$3^2 + 4^2 = 5^2$$

$$12^2 + 5^2 = 13^2 \text{ etc.}$$

It was merely geometry-oriented algebra. The process of dealing with and investigating unknown quantities dates back to *Sthānāṅga-sūtra* (325 B.C.) in which we find the term *jāvantāvatī* (= *yāvattāvat*) in *sūtra* 747. In fact, *Sthānāṅga-sūtra* finds the mention of the following: simple equations

(*yāvattāvat*), quadratic equations(*varga*), cubic equations (*ghana*), bi-quadratic equation (*vargavarga*), permutations and combinations (*vikalpa*). Further, the *Bakshāli Manuscript* contains *yādṛc*, *iṣṭakarma* (rule of false position). *Iṣṭakarma* is a process of arithmetic in which the answer is supposed to be 1 or 100. Some problems on *iṣṭakarma* in the manuscript cannot be solved without the application of simple equations.[27]

In fact, *iṣṭakarma* is considered by some mathematicians to be the root of *Bījagaṇita*.

In the period of Āryabhaṭa, the term *gulikā* was employed for an unknown quantity and thus the following verse of Āryabhaṭa is considered to be the explicit foundation of *Bījagaṇita*:

gulikāntareṇa vibhajeddvayoḥ puruṣayostu rūpakaviśaṣam/ labdhaṃ gulikāmūlyaṃ yadyarthakṛtam bhavati tulyam//[28]

According to Bhāskara I (A.D.629) the word *gulikā* is used for a thing of unknown value. The term *yāvattāvat* is also used for such a thing: *rūpaka* is used for a coin. The verse has thus the meaning: If two persons have equal wealth, then in order to get the value of one *gulikā* divide the difference between the *rūpakas* (known amounts) with the two persons by the difference between their *gulikās*. The quotient will then be the desired value. In explicit words the verse says: Two persons have equal wealth. One of them has m *gulikās* and r *rūpakas*. The verse tells us the process of finding the cost of one *gulikā* in terms of coins (*rūpakas*). Thus algebraically: if $m x + r = n x + p$ where x is the cost of one *gulikā*, then $x = \frac{p-r}{m-n}$ coins.

Parameśvara (A.D. 1430), while explaining the process given in this verse, has given the following example: Two businessmen have equal wealth. One of them has 100 rupees and six cows and the other has 60 rupees and eight cows. What is the price of a cow? Thus, we have the equation

$$100 + 6x = 60 + 8x$$

whence $x = 20$.

In Sanskrit literature *Bījagaṇita* has mainly two synonyms: (i) *Kuṭṭakagaṇita*, (ii) *Avyaktagaṇita*. In the *Brāhma-sphuṭa-siddhānta* of Brahmagupta, there is a separate chapter on *Bījagaṇita* named as *Kuṭṭakādhyāya*. It is evident that by the time of Brahmagupta (A.D.598 – 665) the rules for algebraic operations appeared as axioms.

Caliph Al-Ma'mūn, a ruler of Baghdād during A.D. 813-833, established a house of wisdom, named *Bayt-al-Hakmā*. In this period Muhammed

ibn al-Khwārizmī went to Afghanistan on an administrative assignment and while returning home he visited India. After return, he wrote a book in A.D.830 called *Hisāb al-jabra w'al-maqābalā* and dedicated it to the Caliph.

The book is either a translation of works of Brahmagupta or Bhaṭṭa Balabhadra or else, it is based on them, as is indicated by the *Encyclopaedia Britannica* :

> 'The circumstances of this treatise professing to be only a compilation and moreover, the first Arabian work of the kind... An Italian merchant, carried it to Italy. He is reported to have written *Scritti di Leonardo Pisano*, 2 volumes, ed. B. Bonocompagni, Rome (1857, 1862). It was he, due to whom *al-jabra w'al-muqabala* could reach Europe. It was translated twice into Latin in the 12th century by Gerard of Cremona and Robert of Chester (A.D. 1140) as *Liber algebrae almucabala* and gained there such a popularity that the subject itself was named "algebra" which is an abreviated form of "al-jabra w' al-muqabala".'[29]

While pointing out the fact that the Arabs could know the negative quantities at a much later stage Cressley writes: 'It is curious that this aspect of Hindu algebra did not pass to the Arabs... Nevertheless, it is generally accepted that algebra including the radical solution of quadratic equations, was transmitted from the Hindus to the Arabs before the ninth century had passed.'[30]

Conclusion

It is well-known that the early literature of Indians are the Vedas, various scriptures like Purāṇas, Śāstras, Vedāṅgas, Smṛtis and various sūtras, etc.—all in Sanskrit. A scientific scrutiny of the Vedas and other ancient Indian treatises reveals that these are a marvellous treasure-house of valuable information about the advances made by ancient Indian scholars in various sciences. But we face a difficulty; usually Sanskritists don't know scientific theories and modern scientists do not know Sanskrit.

It is really surprising that a country with such a rich inheritance, has no centre for History of Science, in particular for History of Mathematics, whereas in Moscow there is a large centre exclusively devoted to the History of Science, named Institute of Science and Natural Tecnology.[31] It has separate departments of histories of Mathematics, Physics, Chemistry, Geological Sciences and Engineering. It is remarkable that the Institute was also directed to establish close contacts with scientists of Academy's various other institutions, the Republican Academy of Sciences and the

country's colleges and universities. We should feel ashamed that many Indian mathematicians have not even heard of *Śrīyantra*, whereas this Institute of Moscow recently held a seminar on 'Indian *Śrīyantra* and its Mathematical Characteristics'. It is important to note that the Uzbek Academy of Science at Tashkent has experts on science, like Professor I.M. Hashimov, who are also associated with the Institute of Oriental Studies. Not only this, Professor M.S. Asimov, President of the Tazik Academy of Science, Institute of History at Dushanbe, a great scholar of History of Science, is also the Director of Institute of Oriental Studies at Dushanbe.

Nowadays there is a tremendous growth in various branches of science and in specialization of various topics. Hence a centre in our country is sorely needed to study the history of the various branches of science with particular attention to the Indian heritage. It should have a full-fledged department on History of Mathematics. The mathematics departments of every university in our country should have a compulsory course on the history of mathematics. Only then shall we know how the present mathematics has evolved and what India's contribution was to it.*

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* Most of the content of this talk is from the work done by the author, when he was a fellow of the Indian Institute of Advanced Study, Rashtrapati Nivas, Simla, during 1985-87.

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वैदिक गणित की उपादेयता

दिला राम

यह गणित अथर्ववेद के परिशिष्ट-1 में वर्णित 16 सूत्रों एवं 16 उपसूत्रों पर आधारित है।[1] ये सूत्र एवं उपसूत्र इतने सूक्ष्म एवं गूढ़ हैं कि इनके शाब्दिक अर्थ समझ लेने पर भी गणित से उनका सम्बन्ध स्थापित करना अपने आप में एक दुरूह कार्य है। कुछ वेदज्ञ, जिनमें कोलब्रुक, विल्सन एवं ग्रिफिथ के नाम उल्लेखनीय हैं, जब इन्होंने इनका सम्बन्ध गणित से स्थापित करने में अपने को असमर्थ पाया तो इन सूत्रों को इन्होंने "वेदों में त्रुटि", "समझ से बाहर", "बकवास" तथा "निरी मूर्खता" के विशेषणों से परिभाषित किया। गोवर्द्धन-पीठ, जगन्नाथ पुरी के जगद्गुरु शंकराचार्य स्वामी भारतीकृष्ण तीर्थ को कोटिशः धन्यवाद जिन्होंने अपने अथक परिश्रम एवं चिन्तन-मनन के द्वारा वैदिक गणित सूत्रों की सहायता से इस चमत्कारिक गणित का 'नवोदय' किया है। गणित के क्षेत्र में यह गणित एक क्रान्ति का सूत्रपात करेगा। जितना शीघ्र गणित संसार इसे स्वीकार कर लेगा उतना ही मानवता का लाभ होगा।

यह गणित अधिकांश प्रश्नों का उत्तर एक पंक्ति में देने में सक्षम है। अतः यह एक पंक्तीय गणित है। किन्तु कुछ स्थानों पर ही स्मरणार्थ अथवा विद्यार्थियों को आरंभ में समझाने हेतु ही, यदा-कदा दूसरी पंक्ति की आवश्यकता पड़ सकती है। इसकी विधियाँ सहज, सरस एवं मनोरंजक तो हैं ही साथ ही साथ शीघ्र ग्राह्य एवं द्रुतगामी भी हैं। अतः स्वामी जी ने इसे अश्रुविहीन गणित भी कहा है। वास्तव में शीघ्र परिणाम प्राप्ति हेतु शून्य से नौ तक के अंकों अथवा उनसे निर्मित संख्याओं एवं उनके प्रतिरूपों पर की गयी संक्षिप्त से संक्षिप्त गाणनिक क्रियाओं एवं गाणनिक तथ्यों के विश्लेषण को गणित कहते हैं। उदाहरणार्थ एक संख्या को बार-बार जोड़ने से भी गुणा हो सकता है, भाग बार-बार घटाने से सम्भव है, योग वस्तुओं को एक-एक कर गिनने तथा व्यवकलन एक-एक वस्तु उठाने से भी सम्पन्न हो सकता है। पर वह गणित नहीं कहा जा सकता है। वैदिक गणित संक्षिप्तीकरण एवं सरलीकरण गाणनिक क्रियाओं का मार्ग प्रशस्त करता है। इसकी अधिकांश विधियाँ जादुई परिणाम देती हैं। अतएव इसमें कोई अतिशयोक्ति दृष्टिगोचर नहीं होती कि यह गणित वहाँ से आरम्भ होता है, जहाँ प्रचलित गणित समाप्त हो जाता है।

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[1] स्वामी भारतीकृष्ण तीर्थ का लेख, भवन्स जर्नल (अंग्रेजी), दीपावली विशेषांक 1987 देखिए।

इस गणित की दो प्रमुख मान्यताएँ हैं, प्रथम मान्यता है कि यह गणित चार के स्थान पर दो ही मौलिक क्रियाएँ मानता है—योग एवं व्यवकलन। द्वितीय मान्यता है कि अंकों में समस्त क्रियाएँ, मात्र योग वियोग द्वारा द्रुतगति से सम्पन्न करने की आन्तरिक क्षमता निहित है। किसी बाह्य शक्ति, जैसे गुणनताओं की आवश्यकता नहीं पड़ती। रामानुज, भास्कराचार्य आदि भारतीय गणितज्ञ तो इन अंकों को ईश्वर का प्रतीक मानते हैं। इस आधार पर अंक ज्योतिष का आविष्कार हुआ है तथा सरस्वती-पूजन में इन अंकों के द्वारा पूजा करने का तंत्रों में विधान है।

वैदिक गणित के सूत्रों एवं उपसूत्रों के आधार पर इतनी विधियाँ खोजी जा सकती हैं कि उन्हें देखकर आश्चर्य होने लगता है। यदि वैदिक गणित तथा प्रचलित गणित दोनों का समन्वय स्थापित कर लिया जाय तो और भी विस्मयकारी विधियाँ प्रकाश में आती हैं। इस प्रकार कितनी ही दीर्घकीय संख्याओं पर क्रियाएँ करना अत्यन्त सरल हो जाता है। वस्तुतः समन्वय अपरिहार्य है, क्योंकि विद्यार्थियों को दोनों ही गणित सीखना लाभदायक है। प्रचलित गणित को त्यागकर वैदिक गणित पर ही निर्भर करना, वैदिक गणित के प्रचार एवं उसे स्वीकार कराने का मूल उद्देश्य नहीं है।

वैदिक गणित में परिणामों की अनेकों जाँच विधियाँ खोज ली गयी हैं। अतएव इसमें उत्तरमाला की आवश्यकता नहीं पड़ती है। जाँच प्रक्रियाएँ इस गणित की अद्भुत विशेषताएँ हैं।

वैदिक गणित की विधियाँ केलकुलेटरों एवं कम्प्यूटरों से भी उत्कृष्ट एवं विश्वसनीय परिणाम देती हैं। आधुनिक कार्यालयों में कम्प्यूटरों द्वारा तैयार परिणाम बहुत बार अशुद्ध होते हैं तथा दुकानदारों आदि के द्वारा केलकुलेटरों से लगाये गये हिसाबों की भी यही दशा है। पुनश्च केलकुलेटरों एवं कम्प्यूटरों में अशुद्धियों का परीक्षण करने का कोई उपबन्ध नहीं है। अनेक बार दीर्घकीय संख्याओं पर क्रियाएँ करना सामान्य केलकुलेटरों के द्वारा सम्भव नहीं है। अतएव केलकुलेटर एवं कम्प्यूटर हमारे महान सहायक होते हुए भी विश्वस्तता की दृष्टि से उनकी उपादेयता सन्देहास्पद है। इन विधियों के द्वारा व्यक्तिगत मानसिक कुशाग्रता का ध्यान रखकर अंक गणित की गणनाएँ लगभग केलकुलेटरों की गति से ही सम्पन्न करने की क्षमता प्राप्त हो जाती है। सरल एवं संक्षिप्त विधियाँ होने के कारण अशुद्धियों की संभावना कम रह जाती है। जाँच प्रक्रिया लागू करने पर तो अशुद्धियों की संभावना शून्य ही रह जाती है।

वैदिक गणित आधुनिक शिक्षा-शास्त्र के मूलभूत सिद्धान्त "खेल द्वारा पढ़ाओ" को पूर्णतया प्रतिपादित करता है। ये विधियाँ इतनी रुचिकर हैं कि बच्चे इन्हें मानसिक खेल ही समझते हैं। वे गणित की प्रक्रियाओं को मानसिक भार समझकर उकताते नहीं, अपितु

खेल जैसा आनन्द लेकर और अधिक गणित सीखने का प्रयास करने लगते हैं। प्रायः बच्चे इन विधियों का अवलोकन कर अवाक् रह जाते हैं। लेखाकार, सांख्यिक, अभियांत्रिक, गणितज्ञ आदि केलकुलेटरों एवं कम्प्यूटरों की उपादेयता में सन्देह करने लगते हैं। इस गणित की जाँच विधियों से परिणामों की जाँच करके तो वे वैदिक गणित का उपकार मानने लगते हैं।

इस गणित में प्रक्रियाओं को सम्पन्न करने की विधियों की विविधता होने के कारण रुचि अनुकूल विधि चुनने की सुविधा भी प्राप्त है। इन सूत्रों में इतनी शक्ति निहित है कि और भी अधिक विधियाँ खोजी जा सकती हैं। विधियों की बहुलता होने पर भी विधियों को स्मरण रखना कठिन नहीं है। नियमित रूप से गणित पढ़ते समय इन विधियों के आधार एवं प्रमाण भी पढ़ाये जायेंगे। आधार इतने थोड़े हैं कि उन्हें स्मरण रखना कोई कठिन नहीं है।

शिक्षण संस्थाओं में गणित पढ़ाने के अनेकों उद्देश्यों में से एक उद्देश्य विद्यार्थियों में मौलिक चिन्तन एवं तर्कशक्ति का विकास करना भी है। वैदिक गणित के सूत्रों के आधार पर कुछ काल पश्चात् छात्र स्वयं भी नई-नई विधियाँ खोजने में प्रवृत्त होने लगेंगे। उनकी यह प्रवृत्ति अन्य विषयों में भी मौलिक रूप से सोचने की प्रवृत्ति को जागृत करेगी। मौलिक चिन्तन एवं तर्कशक्ति का विकास वैदिक गणित द्वारा आरंभिक कक्षाओं से स्वतः ही प्रस्फुटित होने लगता है। प्रचलित गणित में इतना संभव नहीं है। उसकी विधियाँ इतनी घिसी-पिटी एवं दीर्घ हैं कि विद्यार्थियों में मौलिक चिन्तन की प्रवृत्ति अपेक्षाकृत कम जागृत होती है।

'एकाग्रता' की गणित शिक्षा में अत्यधिक आवश्यकता है, परन्तु वैदिक गणित में अपेक्षाकृत और भी अधिक है। यद्यपि वैदिक गणित की विधियाँ प्रचलित गणित से सरल एवं द्रुतगामी हैं, परन्तु 'एकाग्रता' की आवश्यकता अधिक पड़ती है। उदाहरणार्थ गुणन-तन्त्रिकाओं के द्वारा गुणा-भाग की क्रियाएँ सम्पन्न करने में कण्ठस्थ तालिकाओं की पुनरावृत्ति करने में कम 'एकाग्रता' की आवश्यकता पड़ती है। अतः अशुद्धियों की संभावना भी अधिक है। वैदिक गणित में क्योंकि क्रियाएँ योग-वियोग द्वारा मन में सोचकर उसी समय करनी पड़ती हैं, अतः 'एकाग्रता' की आवश्यकता अधिक है, परन्तु साथ ही अशुद्धियों की सम्भावना कम है। 'एकाग्रता' की आवश्यकता अधिक होने पर भी मस्तिष्क पर चिन्तन भार कम पड़ता है, क्योंकि योग-वियोग की क्रियाएँ गुणन-तालिकाओं से सरल हैं। गुणन-तालिकाओं का बड़ी संख्याओं तक सम्बन्ध पड़ता है, जबकि योग-वियोग का अंकों से अथवा बहुत ही छोटी संख्याओं से।

निम्न कक्षाओं में बाल-मनोविज्ञान के अनुसार शिक्षा बाल-केन्द्रित होनी चाहिए। विषय-केन्द्रित नहीं। इसमें अध्यापक को बच्चों को उनके मानसिक स्तर के अनुसार अपने साथ लेकर चलना चाहिए। फलतः व्यक्तिगत ध्यान देना आवश्यक है। वैदिक गणित में अधिकांश कार्य मौखिक ही सम्पन्न होने के कारण विद्यार्थियों पर व्यक्तिगत ध्यान और भी आवश्यक है। तथापि यदि कक्षा में विद्यार्थियों की संख्या अधिक होने के कारण अध्यापक व्यक्तिगत ध्यान देने में कठिनाई का अनुभव करे तो भी वैदिक गणित की विधियाँ विद्यार्थी सरलता से ही समझ लेंगे। इस गणित में बड़ी और छोटी, दोनों ही प्रकार की कक्षाओं में परिस्थितियों के अनुसार सामंजस्य स्थापित करने की क्षमता निहित है।

प्रचलित गणित बच्चों में कंठस्थ करने की प्रवृत्ति को प्रोत्साहित करता है। फलतः बच्चे गणित सीखने से ही जी चुराने लगते हैं। परन्तु वैदिक गणित में क्योंकि चिन्तन भार कम पड़ता है, अतः कंठस्थ करने की प्रवृत्ति पर अवरोध उत्पन्न होता है और मंद बुद्धि बच्चे भी गणित में कुशाग्रता प्राप्त करते हैं।

परीक्षाएँ कम से कम समय में, अधिक से अधिक कार्य-सम्पन्नता तथा प्रश्नों का अधिक से अधिक शीर्षकों एवं उपशीर्षकों पर प्रसारण के सिद्धान्तों पर आधारित है। वैदिक गणित में क्योंकि थोड़े समय में अधिक कार्य सम्पन्न होने की क्षमता निहित है, अतः प्रश्न-पत्रों के रचयिता परीक्षार्थियों को अधिक प्रश्न देकर उपेक्षाकृत थोड़े समय में ही अधिक पाठ्यक्रम के ज्ञान का मूल्यांकन कर सकेंगे। इस प्रकार यह गणित विद्यार्थियों के 'चयनित अध्ययन' की प्रवृत्ति पर अवरोध उत्पन्न करने में सक्षम है।

वैदिक गणित में परिणामों की जाँच की अनेकों विधियाँ विद्यमान होने के कारण परीक्षकों का भार हल्का हो जाएगा। विद्यार्थी स्वयं भी अपने उत्तर की सत्यता को परख सकेंगे। वस्तुनिष्ठ प्रश्न-पत्रों में तो यह गणित वरदान स्वरूप ही है।

आजकल गणित के परीक्षा-फल प्रायः बहुत ही असंतोषजनक रहते हैं। इससे सरकार, समाज तथा माता-पिता के धन का अपव्यय एवं विद्यार्थियों के समय तथा शक्ति नष्ट होती हैं। वैदिक गणित को अपनाने से, गणित विषय में परीक्षा फल अधिक संतोषजनक रहने लगेंगे, क्योंकि वैदिक गणित बच्चों में गणित के प्रति रुचि उत्पन्न करता है।

महान गणितज्ञ श्रीनिवास रामानुजन् ने "संख्या सिद्धान्त" पर चार हजार से भी अधिक सूत्रों का अन्वेषण कर गणित के क्षेत्र में भारत का मस्तक ऊँचा किया है। परन्तु उनकी खोजें शुद्ध गणित से सम्बन्ध रखती हैं। सामान्य जीवन में उनका उपयोग नगण्य है। यद्यपि कैलकुलेटरों एवं कम्प्यूटरों में अब उनके गणित से सहायता ली जा रही है। भास्कराचार्य ने भी "लीलावती" में बहुत संक्षिप्त विधियाँ दी हैं। गणित-जगत इन दोनों ही गणिताचार्यों

की खोजों से ऋणमुक्त नहीं हो सकता है। वैदिक गणित की प्रक्रियाएँ इन दोनों ही विधियों से संक्षिप्त एवं दैनिक जीवन में अधिक उपयोगी हैं।

भारतीय संस्कृति का आधार ग्रंथ "वेद" है। उसे विद्वान संसार के समस्त धर्मों के मूलभूत सिद्धांतों का आदिस्रोत भी स्वीकार करते हैं। वैदिक गणित विद्यार्थियों में वेदों के प्रति आस्था एवं आध्यात्मिकता के प्रति प्रीति उत्पन्न करेगा। इस प्रकार बच्चों में चरित्र-निर्माण करने में पर्याप्त सहायता मिलेगी, जिसका कि आजकल शिक्षण संस्थाओं में अभाव है। प्राकृतिक विज्ञानों का विकास एवं सूक्ष्म कलाओं की प्रगति संस्कृति के आधार स्तम्भ हैं। इन दोनों के ही विकास का आधार गणित है। अतः वैदिक गणित संस्कृति के विकास की गति को तीव्रतर करेगा। 'सर्व धर्म समभाव' की प्रवृत्ति समाज में गति प्राप्त करेगी।

आजकल हम भारतीयों में एवं विशेषकर भारतीय विद्यार्थियों में यह धारणा पुष्ट हो गई है कि आधुनिक पश्चिमी ज्ञान एवं सभ्यता के सम्मुख भारतीय ज्ञान एवं सभ्यता तुच्छ है। इस धारणा के कारण पश्चिमी वर्चस्व को हम आँख मूँदकर स्वीकार कर लिये हैं। परिणामतः गणित की जिन खोजों को भारतीय गणितज्ञ पश्चिम से पर्याप्त समय पहले ही खोज चुके थे, वे आज भी पश्चिम वालों के नाम से ही जगद्-विख्यात हैं। वैदिक गणित हमारे इस अपने ऊपर ही अन्याय के विचार प्रवाह में अवरोध उत्पन्न कर इसके विपरीत सोचने पर विवश कर सकेगा। इस प्रकार वैदिक गणित बच्चों में चरित्र निर्माण, भारतीयता की भावना उजागर कर राष्ट्रीय एकता एवं देशभक्ति की भावना का बीजारोपण कर सकेगा।

वैदिक गणित बढ़ाओ,
ज्ञान विज्ञान बढ़ाओ।
वेद नहीं पूजा-पाठ विधान,
वेद हैं सर्वज्ञान की खान।
वैदिक गणित अपनाओ,
केलकुलेटरों कम्प्यूटरों से मुक्ति पाओ।
यह वैदिक गणित महान्
बढ़ाता वेदों का सम्मान।
वैदिक गणित की विधियाँ आसान,
कि इनमें मस्तिष्क होता नहीं परेशान।

Planetary Theory in the Pañcasiddhāntikā

GEORGE ABRAHAM

Introduction

The *Pañcasiddhāntikā* was written by Varāhamihira in the sixth century A.D. He summarized the different mathematical methods used by Indian astronomers at that time. In this paper, we are concerned with (1) the epicycle model, and (2) algebraic formulae, which describe the motion of the sun, the moon and the planets.

The epicycle model: In this model, the epicycle is a circle whose centre C moves on another larger circle called the deferent, whose centre E is the earth, the planet P moves on the epicycle (Fig. 1a). It is easy to show that it is a transformation of the heliocentric theory, in which the planets move in circular orbits round the sun C. Fig. 1b shows an inner planet. In both figures.

$$\overrightarrow{EP} = \overrightarrow{EC} + \overrightarrow{CP}$$

For an outer planet, the heliocentric picture is in Fig. 2a, and the corresponding epicycle model is in Fig. 2b, where ECPS is a parallelogram. Then

$$\overrightarrow{EP} = \overrightarrow{ES} + \overrightarrow{SP} \qquad \text{in Fig. 2a}$$

$$= \overrightarrow{EC} + \overrightarrow{CP} \qquad \text{in Fig. 2b}$$

We must note that for an outer planet, the radius of the deferent is the distance of the planet from the sun, and the radius of the epicycle is the distance of the earth from the sun. Since the orbits in the heliocentric model are not circular, the epicycle theory was not sufficient to account for planetary motions. To describe this second anomaly, the eccentric model was used in this model, shown in Fig. 3a, in which the earth is situated at a point E away from the centre O so that uniform motion along the circumference produces a non-uniform angular motion for the observer at E. Fig. 3b shows the equivalent epicycle model. The equivalence is expressed by the equations:

$$\vec{EP} = \vec{EO} + \vec{OP} \qquad \text{in Fig. 3a}$$

$$= \vec{EC} + \vec{CP} \qquad \text{In Fig. 3b}$$

The planetary theory in the *Pañcasiddhāntikā* has two epicycles on the same deferent, and the motion is therefore a function of two independent variables. The difficult problem of computation is simplified by an approximation procedure, given also in the *Āryabhaṭīya.* A geometrical explanation of this procedure has been given by Neugebauer. This Indian theory was transmitted to the Arab countries.

The epicycle and eccentric models were also the bases of Ptolemy's planetary theory. However, his model and the computational procedure are different from those of the *Pañcasiddhāntikā.* A detailed comparison of the two models is desirable.

I would like to mention two interestng mathematical topics of the *Pañcasiddhāntikā* theory. One is the formula for the daily motion. For the lunar theory, this is given in chapter 9, verse 13. The formula for planetary motion is not found in the *Pañcasiddhāntikā,* but has been given by later astronomers from this formula, from which the criterion for stationary points can be deduced. Essentially the same formula was derived geometrically by the Greek astronomer Apollonius in the third century B.C.

The second interesting mathematical topic is the trigonometric procedure for calculating the longitudinal and latitudinal components of parallax, found in verses 17 to 25 of chapter 9. Basically the same rules occur in the parallax theory of Kepler, a thousand years later.

Algebraic theories : The epicycle model was preceded by arithmetical or algebraic method, which are also recorded in the *Pañcasiddhāntikā* because some astronomers found them easier to follow. Chapter 2 gives the lesser theory, in which the longitude is a quadratic function of time. Similar formulae in Chapter 17 describe the motion of Saturn and Jupiter.

The daily variations of longitude on linear functions very closely resemble the Babylonian models. In fact the underlying period relations in the two models are identical.

Fig. 1a

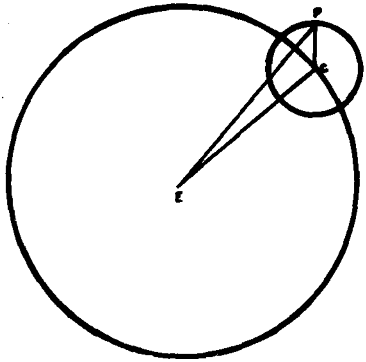


Fig. 1b

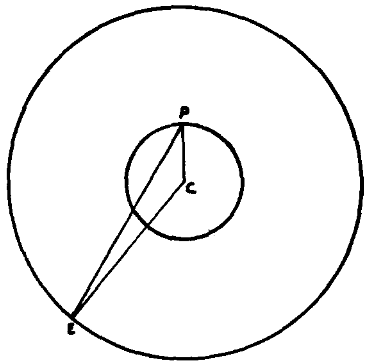


Fig. 2a

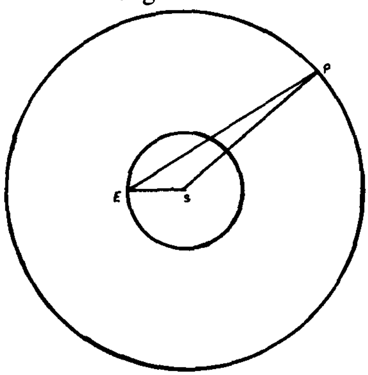


Fig. 2b

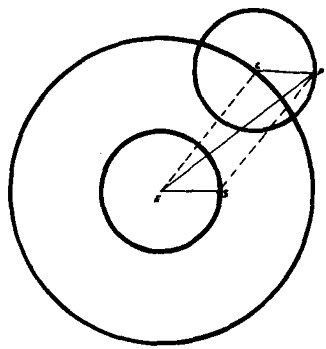


Fig. 3a

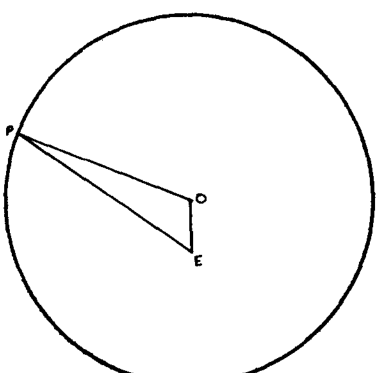
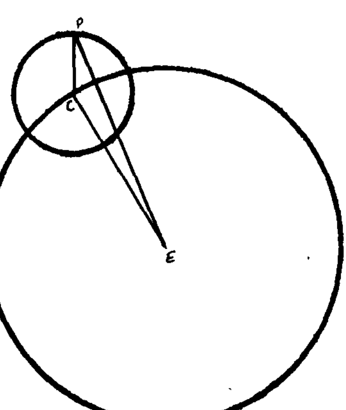


Fig. 3b



Recommendations

H.C. KHARE

The following recommendations have been made keeping in view the views presented by the participants individually during the morning session of 27 March 1988 under the Chairmanship of Shri Ishwarbhai Patel and the recommendations submitted by the three groups constituted for the purpose to the Director.

It is recommended that :

1. A multidisciplinary committee consisting of experts from the fields of mathematics — Vedic, traditional and modern — computer science, the government, and other fields may be formed. The committee will produce the basis of the second workshop which is felt to be necessary before any action is taken to implement Vedic Mathematics (VM) at different levels.

The committee will examine the following issues, either as a full body or through sub-committees, to arrive at the final recommendations on the following issues :

- i. The suitability of VM for introduction at the school level, and all its ramifications, e.g. inclusion in textbooks, training of teachers, logically consistent explanations of VM techniques, teaching aids, etc.
- ii. To study ways and means of encouragement, by which research in VM can be carried out, to understand its potential and limitations at the university level.
- iii. To suggest a multimedia strategy to increase overall awareness of VM and to explore the role of voluntary organizations for this.
- iv. To examine the possibility of providing centralized or regional facilities for training and information dissemination and promoting research in the field of VM.
- v. To write to the Department of Electronics (Government of India) to support development of VM educational software in Indian languages, and to provide financial support to the projects on VM applications in the development of computer technology, such as

knowledge-based algorithms, VM-based computer architecture, etc.

vi. Any other relevant issues raised during the deliberations.

2. The Ministry of Human Resource Development should provide all necessary support to enable effective functioning of the committee.
3. The Ministry should prepare a time-bound programme for preparation of the report of the above committee and organization of goal-oriented workshops.

APPENDIX I

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Appendix II

Excerpts from '*Vedic Metaphysics*'
by H.H. Svāmī Bhāratī Kṛṣṇa Tīrtha
Pages 163 to 167

Gaṇita Sūtras

Now, in dealing with various texts which are to be found in the Vedic literature, I found my first difficulty because the translators who translated these books started with certain notions. I felt that these notions were *a priori* conclusions, conclusions with which they started. And when they started with their conclusion all ready, when they had closed minds, there was no possibility of their going further into the matter and actually finding out the exact meaning of the various texts. That was my first difficulty. Professor Colebrooke and other translators of the ancient Indian scriptural and other texts into English, were meek enough, and modest enough, scientifically-minded enough to say that there were certain portions of the Vedas which were not intelligible. One particular portion I am referring to, a particular portion of the *Atharvaveda* is called the *Gaṇita sūtras*. The *Gaṇita sūtras*, are also called the *Śulba sūtras*, "the easy mathematical formulae", that's the meaning of the expression. And there are sixteen *sūtras*, sixteen aphorisms in all, and the general name 'gaṇita' mathematics, is given to the subject. And then Colebrooke, in his translations, went as far as that, but at that point he stopped and said, "We are unable to understand what the contents of these *sūtras* are, and what connection these sutras have with mathematics." And therefore, there was no possibility of his translating what was absolutely unitelligible to him. But he had the meekness of mind, the scientific frame of mind, to say, "I do not understand. It is unintelligible, it is beyond me." But Horace Hayman Wilson and others of that type, with a superiority complex in which they could not possibly admit that some-thing was unintelligible to them, took another stand altogether. Coming to the same passage, the same portion of the *Atharvaveda*, Horace Hayman Wilson said, "This is all nonsense." That was the single word with which the whole thing was dismissed. And R.T.H. Griffith, the translator of the Ramayana and other poetic works in Sanskrit literature, followed the course. He said it was not merely nonsense but that it was "utter nonsense."

So he improved on his predecessor by adding a very significant and a very dogmatically assertive adjective, utterly nonsensical, no sense can be made out of these matters. Well, that put me on the track. I said there must be something. The ideas may be wrong; the conclusions may be wrong; the details of all the things considered may be wrong; the facts and the factors of the situation taken into account may have been insufficient, and sometimes, irrelevant factors may have been taken into account. And as the natural result of taking irrelevant factors into account and of excluding relevant factors, well, naturally the vision is distorted and the judgement is bound to become warped. The text may be wrong; the conclusion, the arguments may be all wrong, and yet there must be something in the subject which was being discussed with so much earnestness and which the commentators were trying to understand but could make nothing out of, so I went on with my simple idea that there was some meaning.

The grammar, the literature and the figures of speech in Sanskrit give great facility of expressing one's own individual dispositions in a number of different subjects but with the same formula (set of words).

I felt that there must be something like that. And with regard to these formulae, I came to this conclusion, that there must be some kind of key. In king Kaṃsa's reign, famine, pestilence, unsanitary conditions prevailed, that seemed to be the meaning of the text—apparently nothing to do with mathematics, "Mathematical formulae" is the heading of the subject, and inside we are told that the tyrant king Kaṃsa ruled over the people oppressively. He was collecting taxes from them and was doing nothing for them in return, with the result that unsanitary conditions and thefts and all sorts of vices prevailed in his kingdom. That seemed to be an historical account of the king Kaṃsa. But here, the heading is *Gaṇita sūtra* mathe-matical formulae. So I said there must be something. And after long years and years of meditation in the forest, I took the help of lexicographies, lexicons of earlier times, because as a language develops and comes in context with other languages words change their meaning. Words get additional meaning, words get deteriorated in meaning. All sorts of things of this type take place even with regard to one language.

So I studied the old lexicons, including Viśva, Amara, Arṇava, Śabdakalpadruma, etc. books which are known by name only, and which people see in the libraries, but which are never turned for any useful purpose. Well, with these I was able to find out the meanings. I got the key in that way in one instance, and one thing after another helped me in the elucidation of the other *sūtras*, the other formulae. And I found to my

extreme astonishment and gratification that the *sūtras* dealt with mathematics, in all its branches. Only sixteen *sūtras* cover all branches of mathematics—arithmetic, algebra, geometry, trigonometry, physics, plain and spherical geometry, conics, calculus, both differential and integral, applied mathematics of various kinds, dynamics, hydrostatics, statics, kinematics, and all. And, it was a great surprise and a great gratification to me that after all, I was able to get something out of it, and it was not all nonsense as Professor Horace Hayman Wilson and others had declared.

There is renewed interest all over the country in the famous book "Vedic Mathematics", written by the late Shankaracharya of Puri Bharati Krishna Tirthji (1884-1960). Considering the importance of the contents of the book, the Rashtriya Veda Vidya Pratishthan has undertaken a programme of Seminars and Workshops on various issues connected with that book. The first major Workshop was organised by the Pratishthan at Jaipur in association with the Rajasthan University. As it was felt that the papers presented at this Workshop as also the discussions that followed would be of interest to larger sections of teachers, students and general public, it was decided to bring out a publication containing useful material duly edited by Dr. H.C. Khare, Chairman of the Expert Group constituted by the Pratishthan. It is hoped that this publication will provide stimulation for further research.

The Workshop at Jaipur was followed by Workshops or discussions at Ahmedabad, Delhi, Tirupati and Bangalore. Dr. T.S. Bhanumurthy, the former Director of the Ramanujam Institute of Mathematics, Madras, presented proofs of some of the *sutras* contained in "Vedic Mathematics" during the Workshop at Bangalore. These proofs have come to be acknowledged as rational demonstrations of the concerned *sutras* in terms of modern mathematical notations. In the forthcoming book of Dr. Bhanumurthy on Ancient Indian Mathematics, these proofs will form an important part.

Rashtriya Veda Vidya Pratishthan has also commissioned monographs on selected *sutras* contained in the book "Vedic Mathematics".

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ISSUES IN VEDIC MATHEMATICS

Motilal Banarsidass